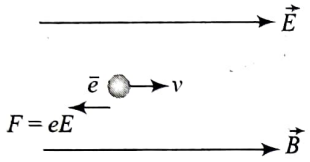
CHAPTER 1

DPP 1.1

Single Correct Answer Type

1. (d) Since electron is moving parallel to the magnetic field, hence magnetic force on it $F_m = 0$.



The only force acting on the electron is electric force, which reduces its speed.

2. (c) $\vec{v} = 2\times10^5\hat{i}$ and $\vec{B} = (\hat{i} + 4\hat{j} - 3\hat{k})$

$$\vec{F} = q(\vec{v}\times\vec{B}) = -1.6\times10^{-19}[2\times10^5\hat{i}\times(i+4\hat{j}-3\hat{k})]$$

$$= -1.6\times10^{-19}\times2\times10^5[\hat{i}\times\hat{i} + 4(\hat{i}\times\hat{j}) - 3(\hat{i}\times\hat{k})]$$

$$= -3.2\times10^{-14}[0 + 4\hat{k} + 3\hat{j}] = 3.2\times10^{-14}(-4\hat{k} - 3\hat{k})$$

$$\Rightarrow \quad |\vec{F}| = 3.2\times10^{-14}\times5 = 1.6\times10^{-13}\ \text{N}$$

3. (c) $r = \dfrac{\sqrt{2mK}}{qB}$ i.e. $r \propto \dfrac{\sqrt{m}}{q}$

Here kinetic energy K and B are the same.

$$\therefore \quad \frac{r_p}{r_\alpha} = \frac{\sqrt{m_p}}{\sqrt{m_\alpha}}\cdot\frac{q_\alpha}{q_p} = \frac{\sqrt{m_p}}{\sqrt{4m_p}}\cdot\frac{2q_p}{q_p} = 1$$

4. (b) This is according to the cross product $\vec{F} = q(\vec{v}\times\vec{B})$ otherwise it can be evaluated by the left-hand rule of Fleming.

5. (a) $r = \dfrac{\sqrt{2mK}}{qB} \Rightarrow K \propto \dfrac{q^2}{m} \Rightarrow \dfrac{K_p}{K_\alpha} = \left(\dfrac{q_p}{q_\alpha}\right)^2 \times \dfrac{m_\alpha}{m_p}$

$$\Rightarrow \quad \frac{1}{K_\alpha} = \left(\frac{q_p}{2q_p}\right)^2 \times \frac{4m_p}{m_p} = 1 \Rightarrow K_\alpha = 1\ \text{MeV}$$

6. (d) $r = \dfrac{\sqrt{2mK}}{qB} \Rightarrow K \propto \dfrac{q^2}{m}$

$$\Rightarrow \quad \frac{K_p}{K_d} = \left(\frac{q_p}{q_d}\right)^2 \times \frac{m_d}{m_p} = \left(\frac{1}{1}\right)^2 \times \frac{2}{1} = \frac{2}{1}$$

$$\Rightarrow \quad K_p = 2\times50 = 100\ \text{keV}$$

7. (b) $r = \dfrac{\sqrt{2mK}}{qB}$ i.e. $r \propto \dfrac{\sqrt{m}}{q}$

Here kinetic energy K and B are the same.

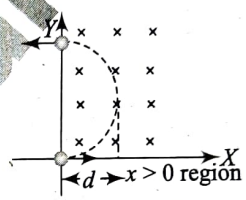
$$\therefore \quad \frac{r_e}{r_p} = \sqrt{\frac{m_e}{m_p}}\times\frac{q_p}{q_e} \Rightarrow \frac{r_e}{r_p} = \sqrt{\frac{m_e}{m_p}} \quad (\because q_e = q_p)$$

Since $m_e < m_p$, therefore $r_e < r_p$

8. (c) $r = \dfrac{1}{B}\sqrt{\dfrac{2mV}{q}} \Rightarrow r \propto \sqrt{\dfrac{m}{q}} \Rightarrow \dfrac{r_x}{r_y} \times \sqrt{\dfrac{m_x}{q_x}\times\dfrac{q_y}{m_y}}$

$$\Rightarrow \quad \frac{R_1}{R_2} = \sqrt{\frac{m_x}{m_y}\times\frac{2}{1}} \Rightarrow \frac{m_x}{m_y} = \frac{R_1^2}{2R_2^2}$$

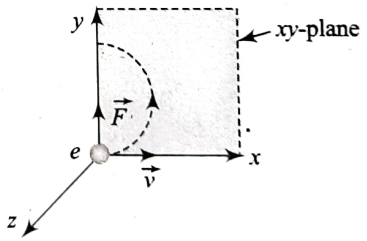
9. (b) It is easy to understand the given problem, along with the following figure.



d = radius of path

$$= \frac{mv}{qB}$$

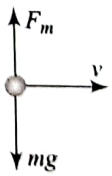
10. (b) To move the electron in xy plane, force on it must be acting in the y-direction initially. The direction of $\vec{F}$ is known, and the direction of v is known, hence by applying Fleming's left-hand rule, the direction of magnetic field is also determined.



11. (c) Particle will move with uniform velocity when its acceleration is zero.

i.e. $|F_m| = mg \Rightarrow mg = qvB$

$$\Rightarrow \quad B = \frac{mg}{qv} = \frac{0.6\times10^{-3}\times10}{25\times10^{-9}\times1.2\times10^4} = 20\ \text{T}$$



12. (b) $\vec{F} = -e(\vec{v}\times\vec{B}) \Rightarrow \vec{F} = -e[v\hat{i}\times B\hat{j}] = evB[-\hat{k}]$

i.e. Force on electron is acting towards negative z-axis. Hence particle will move on a circle in xz-plane.

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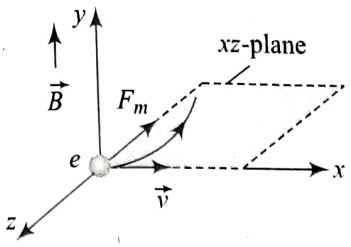

13. (c) By using $r = \dfrac{mv}{qB} = \dfrac{v}{\left(\frac{q}{m}\right)B} \Rightarrow r \propto \dfrac{1}{(q/m)}$

$\because \quad \left(\dfrac{q}{m}\right)_{e^-} > \left(\dfrac{q}{m}\right)_{p^+} > \left\{\left(\dfrac{q}{m}\right)_d = \left(\dfrac{q}{m}\right)_\alpha\right\}$

$\therefore \quad R_d = R_\alpha$

14. (d) Magnetic field produced by wire at the location of charge is perpendicular to the paper inwards. Hence by applying Fleming's left-hand rule, force is directed along *OY*.

15. (c) For no deflection in mutually perpendicular electric and magnetic field $v = \dfrac{E}{B} = \dfrac{3.2\times10^5}{2\times10^{-3}} = 1.6\times10^8$ m/s

If electric field is removed then due to only magnetic field radius of the path described by electron $r = \dfrac{mv}{qB} = \dfrac{9.1\times10^{-31}\times1.6\times10^8}{1.6\times10^{-19}\times2\times10^{-3}}$ = 0.45 m

Comprehension Type

16. (a) $\omega = \dfrac{qB}{m} = \dfrac{1.6\times10^{-19}\times2}{2\times1.6\times10^{-27}} = 10^8 \quad \left(\text{since } qBv = \dfrac{mv^2}{R}\right)$

17. (b) $r = \dfrac{\sqrt{2mE}}{qB}$

$= \sqrt{\dfrac{2\times2\times1.6\times10^{-27}\times100\times1.6\times10^{-19}\times10^6}{1.6\times10^{-19}\times1.6\times10^{-19}}} = 1$ m

18. (d) No. of passes = $\dfrac{100\times10^6\,\text{eV}}{20\times10^3\,\text{eV}}$

19. (c) Oscillation frequency: $\omega = \dfrac{qB}{m}$

For (a):

$\dfrac{eB}{2m} = \dfrac{eB'}{m} \Rightarrow B' = B/2,$ Hence correct

For (b):

$B = \dfrac{2m\omega}{e} = \dfrac{4m\omega'}{(2e)} \Rightarrow \omega = \omega'$ Hence incorrect

For (c):

$B = \dfrac{2m\omega}{e} = \dfrac{m\omega'}{e} \Rightarrow \omega' = 2\,\omega$ Hence correct

For (d):

$\omega = \dfrac{eB}{2m} = \dfrac{2eB'}{4m} \Rightarrow B' = B$ Hence correct

Hence the correct option is (c).

20. (c) $(\text{K.E.})_{\max} = \dfrac{B^2q^2r^2}{2m}$ $\left[\text{Use, } \dfrac{mv^2}{r} = qVB\right]$

DPP 1.2

Subjective Type

1. The equation of motion for the proton is

$$\vec{F} = q\vec{v}\times\vec{B} = q(v_x\hat{i} + v_y\hat{j} + v_z\hat{k})\times B\hat{i} = qB(v_z\hat{j} - v_y\hat{k})$$

$$= m_p\vec{a} = m_p\left[\left(\frac{dv}{dt}\right)\hat{i} + \left(\frac{dv_y}{dt}\right)\hat{j} + \left(\frac{dv_z}{dt}\right)\hat{k}\right]$$

Thus, $\dfrac{dv}{dt} = 0, \dfrac{dv_y}{dt} = \omega v_z, \dfrac{dv_z}{dt} = -\omega v_y,$

where $\omega = eB/m$. The solution is $v_x = v_{0x}$, and $v_y = v_{0y}\cos\omega t$, and $v_z = -v_{0y}\sin\omega t$. In summary, we have

$$\vec{v}(t) = v_{0x}\hat{i} + v_{0y}\cos(\omega t)\hat{j} - v_{0y}(\sin\omega t)\hat{k}$$

2. Equating the magnitude of the electric force ($F_e = eE$) with that of the magnetic force, we obtain $B = E/v \sin\phi$. The field is smallest when the sin ϕ factor is at its largest value; that is, when $\phi = 90°$. Now, we use $K = \dfrac{1}{2}mv^2$ to find the speed:

$$v = \sqrt{\frac{2K}{m_e}} = \frac{2(2.5\times10^3 eV)(1.60\times10^{-19}\,\text{J/eV})}{9.11\times10^{-31}\text{kg}}$$

$$= 2.96\times10^7\ \text{m/s}$$

Thus, $B = \dfrac{E}{v} = \dfrac{10\times10^3\ \text{V/m}}{2.96\times10^7\ \text{m/s}} = 3.4\times10^{-4}T$

The direction of the magnetic field must be perpendicular to both the electric field $(-\hat{j})$ and the velocity of the electron $(+\hat{i})$. Since the electric force $\vec{F}_e = (-e)\vec{E}$ points in the $(+\hat{j})$ direction, the magnetic force $\vec{F}_B = (-e)\vec{v}\times\vec{B}$ points in the $-\hat{j}$ direction. Hence, the direction of the magnetic field is $-\hat{k}$. In unit-vector notation, $\vec{B} = (-3.4\times10^{-4}T)\hat{k}$.

3. (a) For the magnetic field to have an effect on the moving electrons, we need a nonnegligible component of $\vec{B}$ to be perpendicular to $\vec{v}$ (the electron velocity). It is most efficient, therefore, to orient the magnetic field so it is perpendicular to the plane of the page. The magnetic force on an electron has magnitude $F_B = evB$, and the acceleration of the electron has magnitude $a = v^2/r$. Newton's second law yields $evB = m_ev^2/r$, so the radius of the circle is given by $r = m_ev/eB$. The kinetic energy of the electron is $K = \dfrac{1}{2}m_ev^2$, so $v = \sqrt{2K/m_e}$. Thus,

$$r = \frac{m_e}{eB}\sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2m_eK}{e^2B^2}}$$

This must be less than d, so $\sqrt{\dfrac{2m_eK}{e^2B^2}} \le d$, or $B \ge \sqrt{\dfrac{2m_eK}{e^2d^2}}$.

(b) If the electrons are to travel as shown in the magnetic field must be out of the page. Then the magnetic force is toward the center of the circular path, as it must be (in order to make the circular motion possible).

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4. Letting $B_x = B_y = B_1$ and $B_z = B_2$ and using $(\vec{F} = q\vec{v} \times \vec{B})$ and we obtain (with SI units understood)

$$4\hat{i} - 20\hat{j} + 12\hat{k}$$

$$= 2((4B_2 - 6B_1)\hat{i} + (6B_1 - 2B_2)\hat{j} + (2B_1 - 4B_1)\hat{k})$$

Equating like components, we find $B_1 = -3$ and $B_3 = -4$. In summary,

$$\vec{B} = (-3.0\hat{i} - 3.0\hat{j} - 4.0\hat{k})\ T$$

5. Since both electric and magnetic fields are present, the net force on the electron is the vector sum of the electric force and the magnetic force.

The force on the electron is given by $F = -e(\vec{E} + \vec{v} \times \vec{B})$, where E is the electric field, $\vec{B}$ is the magnetic field, and v is the velocity of the electron. The fact that the fields are uniform with the feature that the charge moves in a straight line, implies that the speed is constant. Thus, the net force must vanish.

The condition $\vec{F} = 0$ implies that

$$E = vB = 500 \text{ V/m}$$

Its direction (so that $\vec{F} = 0$) is downward, or $-\hat{j}$, in the "page" coordinates. In unit-vector notation, $\vec{E} = (-500 \text{ V/m})\,\hat{j}$.

6. The force on the charged particle is given by $\vec{F} = q\vec{v} \times \vec{B}$, where q is the charge, $\vec{B}$ is the magnetic field, and $\vec{v}$ is the velocity of the electron.

We write $\vec{B} = B\hat{i}$ and take the velocity of the particle to be $\vec{v} = v_x\hat{i} + v_y\hat{j}$. Thus,

$$\vec{F} = q\vec{v} \times \vec{B} = q(v_x\hat{i} + v_y\hat{j}) \times (B\hat{i}) = -qv_yB\hat{k}.$$

For the force to point along $+\hat{k}$, we must have $q < 0$

The charge of the particle is

$$q = -\frac{F}{v_yB} = -\frac{0.48\ N}{(4.0 \times 10^3 \text{ m/s})(\sin 37°)(0.0050T)}$$

$$= -4.0 \times 10^{-2} \text{ C}$$

Single Correct Answer Type

7. (b) The charge will not experience any force if $|\vec{F}_e| = |\vec{F}_m|$. This condition is satisfied in option (b) only.

8. (c) For undeviated motion $|\vec{F}_e| = |\vec{F}_m|$, which happened when $\vec{v}, \vec{E}$ and $\vec{B}$ are mutually perpendicular to each others.

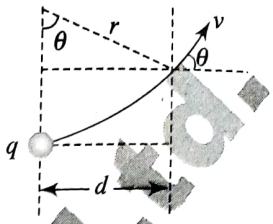
9. (a) Direction of field can be find using Fleming left-hand rule and $r \propto \frac{1}{B}$.

$= \frac{\mu_0}{4\pi} \times \frac{2\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)k^2}{r_1}$. Force between bottom and middle wire

$F_2 = \frac{\mu_0}{4\pi} \frac{\left(\frac{1}{4}\right)\left(\frac{1}{5}\right)k^2}{r_2}$. As the forces are equal and opposite so

$F_1 = F_2 \Rightarrow \frac{r_1}{r_2} = \frac{5}{3}$.

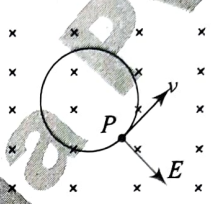
10. (a) From figure it is clear that



$$\sin\theta = \frac{d}{r} \text{ also } r = \frac{p}{qB}$$

$$\therefore \quad \sin\theta = \frac{Bqd}{p}$$

11. (c) When the particle moves along a circle in the magnetic field B, the magnetic force is radially inward. If an electric field of proper magnitude is switched on which is directed radially outwards, the particle may experience no force. It will then move along a straight line with uniform velocity. This will be the case when $qE = qvB \Rightarrow E = vB$



Also $r = \frac{mv}{qB} \Rightarrow v = \frac{qBr}{m}$

So $E = \frac{qB^2r}{m}$

$$= \frac{(10 \times 10^{-6}) \times (0.1)^2 \times 10 \times 10^{-2}}{1 \times 10^{-3} \times 10^{-6}} = 10 \text{ V/m}$$

12. (b) Just before striking the plate, the electric force on the electron is $F_E = eE = eV/d$, in the upward direction. Since the kinetic energy of the electron is $K = \frac{1}{2}mv^2 = \text{eV}$, $v = \sqrt{2}$ eV/m. On the other hand, the magnetic force is

$$F_B = evB = eB\sqrt{\frac{2eV}{m}}$$

in the downward direction. To prevent the electron from striking the plate, we require $F_B = F_E$, or

$$eB\sqrt{\frac{2eV}{m}} > \frac{eV}{d} \Rightarrow B > \frac{V}{d}\sqrt{\frac{m}{2eV}} = \sqrt{\frac{mV}{2ed^2}}$$

13. (a) Using the charge-to-mass ratio is $\frac{q}{m} = \frac{v}{B'r}$. With the speed of the ion given by $v = E/B$, the expression becomes $\frac{q}{m} = \frac{E/B}{B'r} = \frac{E}{BB'r}$.

14. (d) Using and we obtain

$$\vec{F} = q(v_xB_y - v_yB_x)\hat{k} = q(v_x(3B_x) - v_yB_x)\hat{k}$$

where we use the fact that $B_y = 3B_x$. Since the force (at the instant considered) is $F_z\hat{k}$ where $F_z = 6.4 \times 10^{-19}$ N, then we are led to the condition

$$q(3v_x - v_y)B_x = F_z \Rightarrow B_x = \frac{F_z}{q(3v_x - v_y)}.$$

Substituting $v_s = 2.0$ m/s, $v_y = 4.0$ m/s, and $q = -1.6 \times 10^{-19}$ C, we obtatn

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$$B_x = \frac{F_z}{q(3v_x - v_y)}$$

$$= \frac{6.4\times10^{-19}\text{ N}}{(-1.6\times10^{-19}\text{ C})[3(2.0\text{ m/s}) - 4.0\text{ m}]} = -2.0\text{ T}$$

Multiple Correct Answers Type

15. (a, b, d)
Here, force on charged particle due to electric field $\mathbf{F}_E = q\mathbf{E}$.
Force on charged particle due to magnetic field, $\mathbf{F}_m = q$ ($\mathbf{v}$ $\mathbf{E}$ = 0, $\mathbf{B} \neq 0$ $\mathbf{B}$)
Now, $F_E = 0$ if $E = 0$ and $F_m = 0$ if sin $\theta° = 0$ or $\theta° = 0°$ or 180°
Hence, $B \neq 0$
Also $E = 0$ and $B = 0$ and the resultant force
$q\,\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) = 0$. In this *case* $E \neq 0$ and $B \neq 0$

16. (a, c, d)
All are possible because neither mass nor charge is specified.

17. (a, b)
The radius of the circular path is

$$r = \frac{mv}{qB} = \sqrt{\frac{2mK}{qB}}$$

where $K = mv^2/2$ is the kinetic energy of the particle. Thus, we see that $r \propto \sqrt{mK}/qB$.

(a) $\frac{r_d}{r_p} = \sqrt{\frac{m_d K_d}{m_P K_p}\frac{q_p}{q_d}} = \frac{2.0u}{1.0u}\frac{e}{e} = \sqrt{2} \approx 1.4$ and

(b) $\frac{r_\alpha}{r_p} = \sqrt{\frac{m_\alpha K_\alpha}{m_P K_p}\frac{q_p}{q_\alpha}} = \sqrt{\frac{4.0u}{1.0u}}\frac{e}{2e} = 1.0$

Comprehension Type

18. (a) The component of velocity of charged particle along the magnetic field does not change. The component of velocity of charged particle normal to magnetic field only changes in direction but always remains normal to magnetic field. Hence angle between velocity and magnetic field remains the same.

19. (b) $f = \frac{qB}{2\pi m} = \frac{10^3}{19}\times\frac{\sqrt{3800}}{2\pi} = \frac{10^4}{\pi\sqrt{38}}$

20. (d) Pitch = $T.\,V_{\parallel}$

$$= \frac{1}{f}\cdot\frac{\vec{V}\cdot\vec{B}}{|\vec{B}|} = \frac{\pi\sqrt{38}\cdot 400}{10^4\cdot\sqrt{3800}} = \frac{4\pi}{10^3}\text{m} = \frac{\pi}{250}\text{m}$$

DPP 1.3

Subjective Type

1. (a) From symmetry, we conclude that any x-component of force will vanish (evaluated over the entirety of the bent wire as shown). By the right-hand rule, a field in the $\hat{k}$ direction produces on each part of the bent wire a y-component of force pointing in the $-\hat{j}$ direction; each of these components has magnitude

$|F_y| = il\,|\vec{B}|\,\sin 30° = (2.0\text{A})\,(2.0m)\,(4.0T)\sin 30° = 8$ N.
Therefore, the force on the wire shown in the figure is $(-16\hat{j})$ N.

(b) The force exerted on the left half of the bent wire points in the $-\hat{k}$ direction, by the right-hand rule, and the force exerted on the right half of the wire points in the $+\hat{k}$ direction. It is clear that the magnitude of each force is equal, so that the force (evaluated over the entirety of the bent wire as shown) must necessarily vanish.

2. (a) The magnetic force on the wire is $F_B = idB$, pointing to the left. Thus

$$v = at = \frac{F_B t}{m} = \frac{idBt}{m}$$

$$= \frac{(9.13\times10^{-3}\,A)(2.56\times10^{-2}\,m)(5.63\times10^{-2}\,T)(0.0611s)}{2.41\times10^{-5}\text{kg}}$$

$$= 3.34\times10^{-2}\text{ m/s}$$

(b) The direction is to the left (away from the generator).

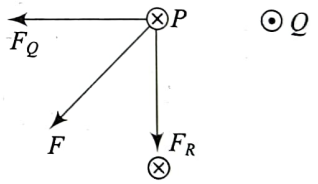
3. We use $d\vec{F}_B = id\vec{L}\times\vec{B}$, where $d\vec{L} = dx\hat{i}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$

$$\vec{F}_B = \int id\vec{L}\times\vec{B} = \int_{x_i}^{x_f} idx\hat{i}\times(B_x\hat{i} + B_y\hat{j}) = i\int_{x_i}^{x_f} B_y dx\hat{k}$$

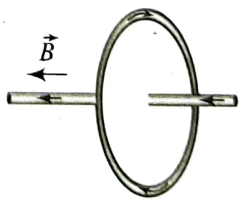
$$= (-5.0)\left(\int_{1.0}^{3.0}(8.0x^2dx)\right)\hat{k} = (-0.35\text{N})\hat{k}$$

Single Correct Answer Type

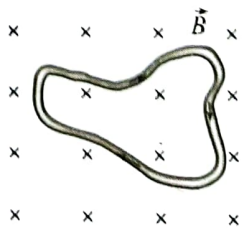
4. (c) The forces F_Q and F_B are the forces applies by wires Q and R respectively on the wire P as shown in figure. Their resultant force F is best shown by C.



5. (b) As shown in the following figure straight wire is placed parallel to the magnetic field produced by circular current. Hence force on wire $F = 0$



6. (a) As shown in figure, since $\vec{L} = 0$



Hence according to $\vec{F} = i(\vec{L}\times\vec{B}) \Rightarrow \vec{F} = 0$

7. (b) On applying Fleming's left-hand rule.

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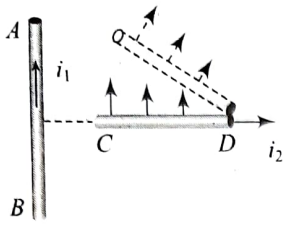
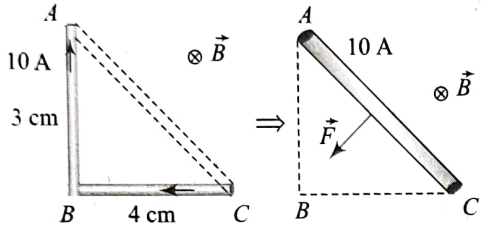
8. (c) Since the force on the rod CD is non-uniform it will experience force and torque. From the left hand side it can be seen that the force will be upward and torque is clockwise.



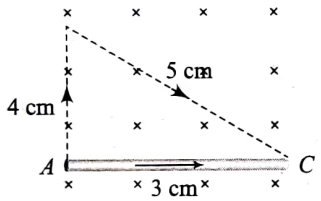
9. (c) According to the question figure can be drawn as shown below.



Force on the conductor ABC = Force on the conductor AC

$= 5 \times 10 \times (5 \times 10^{-2}) = 2.5$ N

10. (b) The given curved wire can be treated as a straight wire as shown

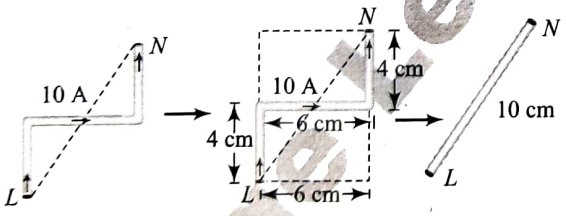


Force acting on the wire AC, $F = Bil = 2 \times 2 \times 3 \times 10^{-2}$

$= 12 \times 10^{-2}$ N along y-axis

So acceleration of wire $= \dfrac{F}{m} = \dfrac{12 \times 10^{-2}}{10 \times 10^{-3}} = 12$ m/s^2

11. (b) The given wire can be replaced by a straight wire as shown below

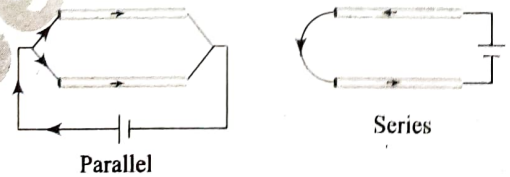


Hence force experienced by the wire

$F = Bil = 5 \times 10 \times 0.1 = 5$ N

12. (c) The given portion of the curved wire may be treated as a straight wire of length $2L$ which experiences a magnetic force $F_m = Bi(2L)$.

13. (a) When connected in parallel the current will be in the same direction and when connected in series the current will be in the opposite direction.

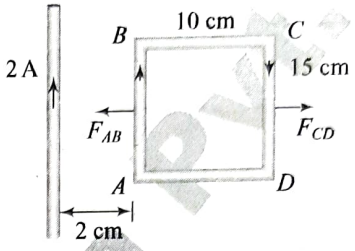


14. (a) Suppose in equilibrium wire PQ lies at a distance r above the wire AB

Hence in equilibrium $mg = Bil \Rightarrow mg = \dfrac{\mu_0}{4\pi}\left(\dfrac{2i}{r}\right) \times il$

$\Rightarrow \quad 10^{-3} \times 10 = 10^{-7} \times \dfrac{2 \times (50)^2}{r} \times 0.5 \Rightarrow r = 25$ mm

15. (a) Force on side BC and AD are equal but opposite so their net will be zero.



But $F_{AB} = 10^{-7} \times \dfrac{2 \times 2 \times 1}{2 \times 10^{-2}} \times 15 \times 10^{-2} = 3 \times 10^{-6}$ N

and $F_{CD} = 10^{-7} \times \dfrac{2 \times 2 \times 1}{(12 \times 10^{-2})} \times 15 \times 10^{-2} = 0.5 \times 10^{-6}$ N

$\Rightarrow \quad F_{net} = F_{AB} - F_{CD} = 2.5 \times 10^{-6}$ N

$= 25 \times 10^{-7}$ N, towards the wire

DPP 1.4

Subjective Type

1. Consider an infinitesimal segment of the loop, of length ds. The magnetic field is perpendicular to the segment, so the magnetic force on it has magnitude $dF = iB\,ds$. The horizontal component of the force has magnitude

$$dF_h = (iB \cos \theta)ds$$

and points inward toward the center of the loop. The vertical component has magnitude

$$dF_y = (iB \sin \theta)ds$$

and points upward. Now, we sum the forces on all the segments of the loop. The horizontal component of the total force vanishes, since each segment of wire can be paired with another, diametrically opposite segment. The horizontal components of these forces are both toward the center of the loop and thus in opposite directions. The vertical component of the total force is

$$F_v = iB \sin\theta \int ds = 2\pi a i B \sin\theta$$

2. (a) The magnetic force must push horizontally on the rod to overcome the force of friction, but it can be oriented so that it also pulls up on the rod and thereby reduces both the normal force and the force of friction. The forces acting on the rod are: $\vec{F}$, the force of the magnetic field; mg, the magnitude of the (downward) force of gravity; $\vec{F}_N$, the normal force exerted by the stationary rails upward on the rod; and $\vec{f}$, the (horizontal) force of friction. For definiteness, we assume the rod is on the

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verge of moving eastward, which means that $\vec{f}$ points westward (and is equal to its maximum possible value μsFN). Thus, $\vec{F}$ has an eastward component F_x and an upward component F_y, which can be related to the components of the magnetic field once we assume a direction for the current in the rod. Thus, again for definiteness, we assume the current flows northward. Then, by the right-hand rule, a downward component (B_d) of $\vec{B}$ will produce the eastward F_x, and a westward component (B_w) will produce the upward F_y. Specifically,

$$F_x = iLB_d,\ F_y = iLB_w$$

Considering forces along a vertical axis, we find

$$F_N = mg - F_y = mg - iLB_w$$

so that $$f = f_{s,\max} = \mu_s(mg - iLB_w)$$

It is on the verge of motion, so we set the horizontal acceleration to zero:

$$F_s - f = 0 \Rightarrow iLB_d = \mu_s(mg - iLB_w)$$

The angle of the field components is adjustable, and we can minimize with respect to it. Defining the angle by $B_w = B\sin\theta$ and $B_d = B\cos\theta$ (which means θ is being measured from a vertical axis) and writing the above expression in these terms, we obtain

$$iLB\cos\theta = \mu_s(mg - iLB\sin\theta)$$

$$\Rightarrow \qquad B = \frac{\mu_s mg}{iL(\cos\theta + \mu_s\sin\theta)}$$

which we differentiate (with respect to θ) and set the result equal to zero. This provides a determination of the angle:

$$\theta = \tan^{-1}(\mu_s)$$

Consequently,

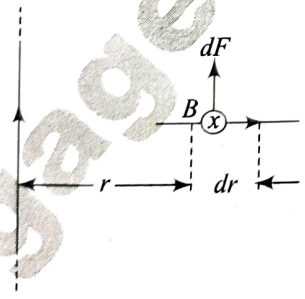
$$B_{\min} = \frac{0.60(1.0\text{ kg})(9.8\text{ m/s}^2)}{(50\text{ A})(1.0\text{ m})(\cos 31^\circ + 0.60\sin 31^\circ)} = 0.10\text{ T}$$

(b) As shown above, the angle is

$$\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.60)$$

3. The force on the current element idl of the conductor is

$$dF = idl\,B\sin 90^\circ$$



$$= idr\frac{\mu_0 i_0}{2\pi r} = \frac{\mu_0 i i_0}{2\pi}\frac{dr}{r}$$

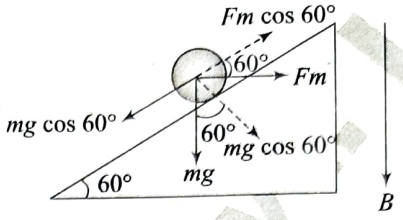
The net force is, $F = \int dF$

$$= \int_{r_0}^{2r_0}\frac{\mu_0 i i_0}{2\pi r}dr = \frac{\mu_0 i i_0}{2\pi}\int_{r_0}^{2r_0}\frac{dr}{r} = \frac{\mu_0 i i_0}{2\pi}\ln 2$$

Single Correct Answer Type

4. (b) Current carrying conductors will attract each other, while electron beams will repel each other.

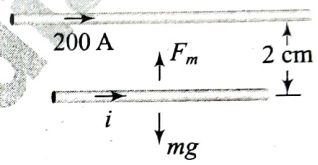
5. (c) The given situation can be drawn as follows



$$F = ilB \Rightarrow mg\sin 60^\circ = ilB\cos 60^\circ$$

$$\Rightarrow \qquad B = \frac{0.01\times 10\times\sqrt{3}}{0.1\times 1.73} = 1\text{ T}$$

6. (c) For floating the second wire



$$\left|\begin{matrix}\text{Downward weight}\\ \text{of second wire}\end{matrix}\right| = \left|\begin{matrix}\text{Magnetic force}\\ \text{on it}\end{matrix}\right|$$

$$\Rightarrow \qquad mg = \frac{\mu_0}{4\pi}\cdot\frac{2i_1 i_2}{a}\times l$$

$$\Rightarrow \qquad \left(\frac{m}{l}\right)g = \frac{\mu_0}{4\pi}\cdot\frac{2i_1 i_2}{a}$$

$$\Rightarrow \qquad 10^{-2}\times 9.8 = 10^{-7}\times\frac{2\times 200\times i}{2\times 10^{-2}} \Rightarrow i = 49\text{ A}$$

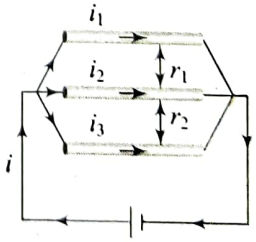
(Direction of current is the same as the first wire)

7. (b) Initially $F_1 + mg + IaB$ (downwards)

when the direction of current is reversed

$$F_2 = mg = -IaB\ (\text{down wards}) \Rightarrow \Delta F = 2IaB$$

8. (c) The wires are in parallel and ratio of their resistances are 3 : 4 : 5. Hence currents in wires are in the ratio $\frac{1}{3}:\frac{1}{4}:\frac{1}{5}$



$$i_1 = \frac{k}{3}$$

$$i_2 = \frac{k}{4}$$

$$i_3 = \frac{k}{5}$$

Force between top and middle wire $F_1 = \frac{\mu_0}{4\pi}\cdot\frac{2i_1 i_2}{r_1}$

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$= \frac{\mu_0}{4\pi} \times \frac{2\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)k^2}{r_1}$. Force between bottom and middle wire

$F_2 = \frac{\mu_0}{4\pi} \frac{\left(\frac{1}{4}\right)\left(\frac{1}{5}\right)k^2}{r_2}$. As the forces are equal and opposite so

$F_1 = F_2 \Rightarrow \frac{r_1}{r_2} = \frac{5}{3}$

9. (d) $F = \frac{\mu_0}{4\pi}\frac{2i_1 i_2}{a}$

$F_1 = \frac{\mu_0}{4\pi} = \frac{2i^2}{x}$ (Attraction)

$F_2 = \frac{\mu_0}{4\pi} = \frac{2i \times 2i}{2x} = \frac{\mu_0}{4\pi}\frac{2i^2}{x}$ (Repulsion)

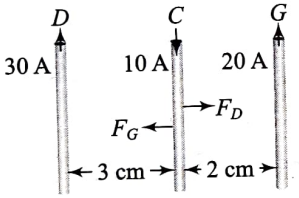
Thus $F_1 = -F_2$

10. (a) For no force on wire C, force on wire C due to wire D = force on wire C due to wire B

$\Rightarrow \quad \frac{\mu_0}{4\pi} \times \frac{2 \times 15 \times 5}{x} \times i = \frac{\mu_0}{4\pi} \times \frac{2 \times 5 \times 10}{(15 - x)} \times l \Rightarrow x = 9\,\text{cm}$

11. (c) Force on wire C due to wire D

$F_D = 10^{-7} \times \frac{2 \times 30 \times 10}{2 \times 10^{-2}} \times 25 \times 10^{-2} = 5 \times 10^{-4}\,\text{N}$ (towards right)



Force on wire C due to wire G

$F_G = 10^{-7} \times \frac{2 \times 20 \times 10}{2 \times 10^{-2}} \times 25 \times 10^{-2} = 5 \times 10^{-4}\,\text{N}$ (towards left)

$\Rightarrow$ Net force on wire C is $F_{net} = F_D - F_G = 0$

12. (a) $\vec{F}_{CAD} = \vec{F}_{CD} = \vec{F}_{CED}$

$\therefore$ Net force on frame $= 3\vec{F}_{CD} = (3)\,(2)\,(1)\,(4)\ (F = ilB)$

$= 24\,\text{N}$.

13. (c) Though it is closed loop, the current is not completing any of the loop.

So, the force will not be zero.

Hence, $F_{loop} = F_{ABC} + F_{AC} + F_{ADC}$

$F_{ABC} = F_{ADC} = F_{AC} = 1\ LB$

Hence, $F_{loop} = 3\ ILB$

14. (b) Since the wire is kept in the uniform field it can be replaced by the straight wire connecting 'O' and 'P'. Also,

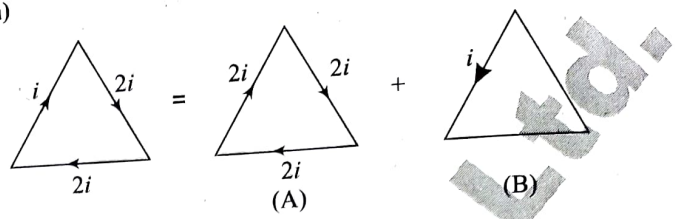
$\vec{L} = (2\hat{i} + 1.5\hat{j})\,\vec{B} = (2\hat{i} + 2\hat{j})$

$\vec{F} = I\vec{L} \times \vec{B} = 10(2\hat{i} + 1.5\hat{j}) \times (2\hat{i} + 2\hat{j})$

$= 10[4 - 3]\hat{k} = 10\hat{k}$

$= 10\,N\hat{k}$

15. (a)



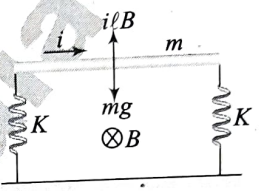
force in figure (A) is zero, and force in figure (B) = i lB.

16. (b) The force on the rod due to magnetic field and gravity is $i\,lB - mg$ (upwards)

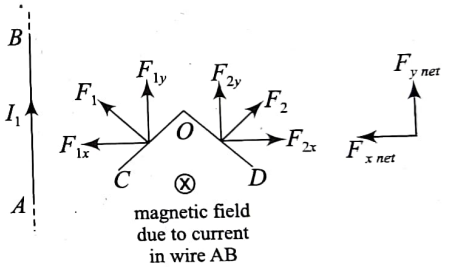
Hence the extension in the springs is $\frac{ilB - mg}{2K}$

(Note that effective spring constant is $2k$)

Therefore the length of the spring is $l_0 + \frac{ilB - mg}{2K}$



17. (d) Magnetic forces F_1 and F_2 act on section CO and OD respectively in direction normal to the wires as shown in figure.



Resolve F_1 and F_2 along x and y axis.

$F_{1x} > F_{2x}$ (wire CO is nearer to AB than OD)

Hence the net force on wire CD has a component along x axis and another component along y-axis.

18. (c) The field $\vec{B}_1$ at point O due to the wire $ABCDE$ is perpendicular to the plane of paper which is along the direction of $i_2\vec{\partial l}$ vector for the second wire.

Hence $i_2\vec{\partial l} \times X\vec{B}_1 = 0$

$\Rightarrow \quad F = B_1 l_2\, l \sin 0 = 0$

i.e. there is no force between the two wires.

Multiple Correct Answers Type

19. (a, c)

$\vec{F}_{BA} = \vec{F}_{CD} = 0$ because magnetic lines are parallel or antiparallel to wire BA and CD respectively.

$\vec{F}_{AD}$ and $\vec{F}_{CB}$ are perpendicular to paper outwards and inwards respectively. These two forces will produce a torque which will rotate the loop in clockwise direction about OO'. The two forces are equal and opposite.

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20. (b,c,d)

Let O and P be the end points of wire. Hence force on current (I) carrying wire placed in uniform magnetic field is $\vec{F} = I(\overrightarrow{OP} \times \vec{B})$.

DPP 1.5

Subjective Type

1. The B_z component will produce a force on this straight segment which points in the $-x$ direction (back toward the hinge) and thus will exert no torque about the hinge. However, the B_x component (which is equal to $B \cos\theta$ where $B = 0.50T$ and $\theta = 30°$) produces a force equal to $F = NiLB$, which points (by the right-hand rule) in the $+z$ direction.

Since the action of the force F is perpendicular to the plane of the coil and is located a distance x away from the hinge, then the torque has magnitude.

$$\tau = (NiLB_x)(x) = NiLxB \cos\theta = (20)$$
$$(0.10\text{ A})(0.10\text{ m})(0.050\text{ m})(0.50\text{ T})\cos 30° = 0.0043\text{ N.m}$$

Since $\vec{\tau} = \vec{r} \times \vec{F}$, the direction of the torque is $-y$. In unit-vector notation, the torque is $\vec{\tau} = (-4.3 \times 10^{-3}\text{ N.m})\,\hat{j}$

An alternative way to do this problem is through the use of $\vec{\tau} = \vec{\mu} \times \vec{B}$. The magnetic moment vector is

$$\vec{\mu} = -(NiA)\hat{k} = -(20)(0.10\text{ A})(0.005\text{ m}^2)\hat{k}$$
$$= -(0.01\text{ A.m}^2)\hat{k}$$

The torque on the loop is

$$\vec{\tau} = \vec{\mu} \times \vec{B} = (-\mu\hat{k}) \times (B\cos\theta\hat{i} + B\sin\theta\hat{k}) = -(\mu B\cos\theta)\hat{j}$$
$$= -(0.01\text{ A.m}^2)(0.50T)\cos 30°\,\hat{j}$$
$$= (-4.3 \times 10^{-3}\text{ N.m})\hat{j}$$

2. (a) The area of the loop is

$$A = \frac{1}{2}(30\text{ cm})(40\text{ cm}) = 6.0 \times 10^2\text{ cm}^2, \text{ so}$$
$$\mu = iA = (5.0)(6.0 \times 10^{-2}\text{m}^2) = 0.30\text{ A.m}^2$$

(b) The torque on the loop is

$$\tau + \mu B\sin\theta = (0.30\text{ A.m}^2)(80 \times 10^3 T)\sin 90°$$
$$= 2.4 \times 10^{-2}\text{ N.m}$$

3. Magnetic forces on a current-carrying coil produce a torque that tends to align the magnetic dipole moment with the field. The magnetic energy of the dipole depends on its orientation relative to the field.

The magnetic potential energy of the dipole is given by $U = -\vec{\mu}\cdot\vec{B}$, where $\vec{\mu}$ is the magnetic dipole moment of the coil and B is the magnetic field. The magnitude of $\vec{\mu}$ is $\mu = NiA$ where i is the current in the coil, N is the number of turns, A is the area of the coil. On the other hand, the torque on the coil is given by the vector product $\vec{\tau} = \vec{\mu} \times \vec{B}$.

(a) By using the right-hand rule, we see that $\vec{\mu}$ is in the $-y$ direction. Thus, we have

$$\vec{\mu} = (NiA)(-\hat{j}) = -(3)(2.00\text{ A})(4.00 \times 10^{-3}m^2)\hat{j}$$
$$= -(0.0240\text{ A.m}^2)\hat{j}.$$

The corresponding magnetic energy is

$$U = -\vec{\mu}.\vec{B} = -\mu_y B_y = -(-0.0240\text{ A.m}^2)$$
$$(-3.00 \times 10^{-3}T) = -7.20 \times 10^{-5}\text{ J}$$

(b) Using the fact that $\hat{j}\cdot\hat{i} = 0$, $\hat{j}\times\hat{j} = 0$, and $\hat{j}\times\hat{k} = \hat{i}$, the torque on the coil is

$$\vec{\tau} = \vec{\mu} \times \vec{B} = \mu_y B_z\hat{i} - \mu_y B_x\hat{k}$$
$$= (-0.0240\text{ A.m}^2)(-4.00 \times 10^{-3}T)\hat{i}$$
$$-(-0.0240\text{ A.m}^2)(2.00 \times 10^{-3}T)\hat{k}$$
$$= (9.60 \times 10^{-5}\text{ N.m})\hat{i} + (4.80 \times 10^{-5}\text{ N.m})\hat{k}$$

The magnetic energy is highest when $\vec{\mu}$ is in the opposite direction of $\vec{B}$, and lowest when $\vec{\mu}$ lines up with $\vec{B}$.

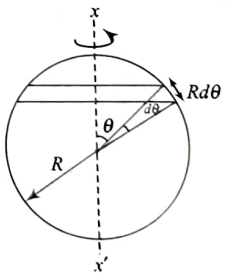
4. Looking at the point in the graph (Fig. (b)) corresponding to $i_2 = 0$ (which means that coil 2 has no magnetic moment) we are led to conclude that the magnetic moment of coil 1 must be $\mu_1 = 2.0 \times 10^{-5}\text{ A.m}^2$. Looking at the point where the line crosses the axis (at $i_2 = 5.0$ mA) we conclude (since the magnetic moments cancel there) that the magnitude of coil 2's moment must also be $\mu_2 = 2.0 \times 10^{-5}\text{ A.m}^2$ when $i_2 = 0.0050$ A, which means

$$N_2A_2 = \frac{\mu_2}{i_2} = \frac{2.0 \times 10^{-5}\text{ A.m}^2}{0.0050\text{ A}} = 4.0 \times 10^{-3}\text{ m}^2$$

Now for the problem let us consider the direction of coil 2's current changed so that the net moment is the sum of two (positive) contributions, from coil 1 and coil 2, specifically for the case that $i_2 = 0.007$ A. We find that total moment is

$$\mu = (2.0 \times 10^{-5}\text{ A.m}^2) + (N_2A_2i_2) = 4.8 \times 10^{-5}\text{ A.m}^2$$

5. The solid sphere rotates about axis xx' as shown in the figure with angular speed ω.



We choose an elementary ring of radius $R\sin\theta$ and width $Rd\theta$. The charge on ring is

$$dq = \sigma(2\pi R\sin\theta)(Rd\theta) \text{ where}$$
$$\sigma = \frac{Q}{4\pi R^2} = \frac{\text{Charge}}{\text{area}} \text{ on the sphere.}$$

Ring rotates about its axis with angular speed ω and hence is equivalent to a current carrying ring.

$\therefore$ Magnetic dipole moment of the ring is

$$dM = \left(\frac{dq}{2\pi}\omega\right)\pi(R\sin\theta)^2 = \omega\sigma R^4\pi\sin^3\theta d\theta$$

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$\therefore$ Magnetic dipole moment of sphere is

$$M = \int_0^{\pi} \omega\, \sigma R^4 \pi \sin^3\theta d\theta = \frac{QR^2\omega}{3}$$

Single Correct Answer Type

6. (d) Couple of force on loop S will be maximum because for the same perimeter the area of loop will be maximum and magnetic moment of loop $= i \times A$. So, it will also be maximum for loop S.

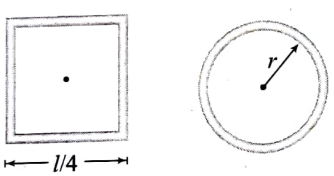
7. (c) Suppose length of each wire is l. $A_{square} = \left(\frac{l}{4}\right)^2 = \frac{l^2}{16}$

$$A_{circle} = \pi r^2 = \pi\left(\frac{l}{2\pi}\right)^2 = \frac{l^2}{4\pi}$$

$\because$ Magnetic moment $M = iA$

$$\Rightarrow \quad \frac{M_{square}}{M_{circle}} = \frac{A_{square}}{A_{circle}}$$

$$= \frac{l^2/16}{l^2/4\pi} = \frac{\pi}{4}$$



8. (b) If the radius of circle is r, then $2\pi r = L \Rightarrow r = \frac{L}{2\pi}$

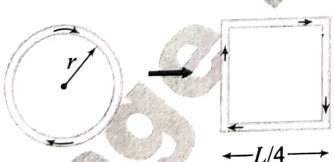
Area $= \pi r^2 = \frac{\pi L^2}{4\pi^2} = \frac{L^2}{4\pi}$

Magnetic moment $= IA = \frac{IL^2}{4\pi}$

9. (d) Initially for circular coil $L = 2\pi r$ and $M = i \times \pi r^2$

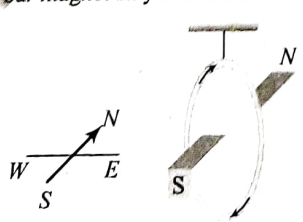
$$= i \times \pi\left(\frac{L}{2\pi}\right)^2 = \frac{iL^2}{4\pi} \qquad \text{(i)}$$

Finally for square coil $M' = i \times \left(\frac{L}{4}\right)^2 = \frac{iL^2}{16}$ (ii)



Solving equation (i) and (ii) $M' = \frac{\pi M}{4}$

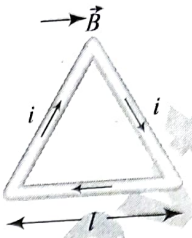
10. (c) Current carrying loop behaves as a bar magnet. A freely suspended bar magnet stays in the N–S direction.



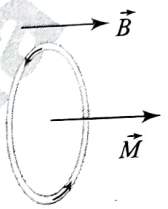
11. (d) Since $\theta = 90°$

Hence $\tau = NIAB = 1 \times I \times \left(\frac{\sqrt{3}}{4} l^2\right) B$

$$= \frac{\sqrt{3}}{4} l^2 B$$

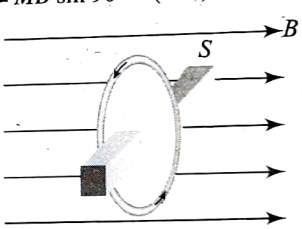


12. (c) In equilibrium angle between $\vec{M}$ and $\vec{B}$ is zero. It is happened when plane of the coil is perpendicular to $\vec{B}$



13. (c) As shown in the following figure, the given situation is similar to a bar magnet placed in a uniform magnetic field perpendicularly. Hence torque on it

$$\tau = MB \sin 90° = (i\pi r^2)B$$



14. (a) $\tau = NB_iA \sin\theta$ so the graph between τ and θ is a sinusoidal graph.

15. (a) The rotation of the loop by 30° about an axis perpendicular to its plane make no change in the angle made by axis of the loop with the direction of magnetic field, therefore the work done to rotate the loop is zero.

16. (a) The magnetic moment of current carrying loop

$$M = niA = ni(\pi r^2)$$

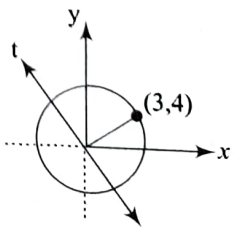
Hence the work done in rotating it through 180°

$$W = MB(1 - \cos\theta) = 2MB = 2(ni\pi r^2)B$$
$$= 2 \times (50 \times 2 \times 3.14 \times 16 \times 10^{-4}) \times 0.1 = 0.1 \text{ J}$$

17. (a) Magnetic moment $\vec{M} = \pi r^2 i\hat{i}$ and $\vec{B} = 3\hat{i} + 4\hat{j}$

$\therefore \quad \vec{\tau} = \vec{M} \times \vec{B} = \pi r^2(3\hat{i} - 4\hat{i})$

$\vec{\tau}$ will be along the direction shown



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Hence, the point about which the loop will be lift up will be: (3, 4)

Comprehension Type

18. (a) The current path *abcdefgha* can be treated as superposition of three loops *fghaf*, *fabef* and *ebcde* , each carrying current *I*.

19. (b) The magnetic field at centre of cube cancels due to two square loops *fghaf* and *ebcde* each carrying current *I*. The magnetic field at centre of cube is only due to loop *fabef*. Hence the magnetic field at centre of cube is in negative *y*-direction.

20. (d) The current path *abcdefgha* can be treated as superposition of three loops *fghaf*, *fabef* and *ebcde* each carrying current *I*. The dipole moment two square loops *fghaf* and *ebcde* each carrying current *I* cancels out. Hence the dipole moment is only due to loop *fabef*. Now the dipole moment of loop is in negative *y*-direction and direction of external magnetic field is in positive *y*-direction. Therefore torque on loop is zero.

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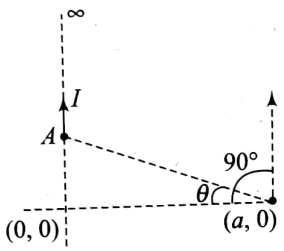
CHAPTER 2

DPP 2.1

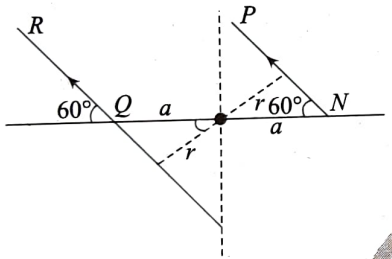
Subjective Type

1. $B = \frac{\mu_0}{4\pi}\frac{i}{a}[\sin 90^0 + \sin(-\theta)] = \frac{\mu_0}{4\pi}\frac{i}{a}(1 - \sin\theta)$

$$= \frac{\mu_0}{4\pi}\frac{i}{a}\left(1 - \frac{b}{\sqrt{a^2+b^2}}\right)$$



2. Due to PQ and MN, magnetic field at O will be zero.



$$r = a\sin 60° = \frac{\sqrt{3}}{2}a$$

$$B_{QR} = \frac{\mu_0}{4\pi}\frac{I}{r}[\sin 90° - \sin 30°]\otimes$$

$$= \frac{\mu_0}{4\pi}\frac{I}{(\sqrt{3}\,a/2)} \times \frac{1}{2}\otimes = \frac{\mu_0 I}{4\pi\sqrt{3}\,a}\otimes$$

$$B_{NP} = \frac{\mu_0}{4\pi}\frac{I}{r}[\sin 90° + \sin 30°]\odot = \frac{3\mu_0 I}{4\pi\sqrt{3}\,a}\odot$$

Net field at O:

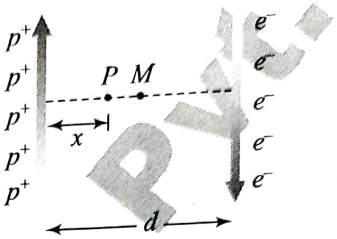
$$B = B_{NP} - B_{QR} = \frac{\mu_0 I}{2\sqrt{3}\,\pi a}\odot$$

Single Correct Answer Type

3. (a) Every point on line AB will be equidistant from X and Y-axes. So magnetic field at every point on line AB due to wire 1 along X-axis is equal in magnitude but opposite in direction to the magnetic field due to wire along Y-axis. Hence B_{net} on $AB = 0$.

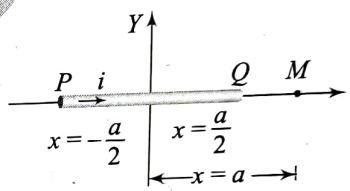
4. (c) The magnetic field at points to the right of the proton beam acts perpendicular to the paper inwards (×). The magnetic field at points to the left of the electron beam acts perpendicular to the paper outwards (·).

Magnetic field at mid point M is zero.



Magnetic field at the points closer to proton beam acts perpendicular to the paper inwards (i.e. (×)) and at the points closer to electron beam it acts outwards, i.e. (·). In the given options graph (*c*) satisfies all the conditions.

5. (d) Magnetic field at a point on the axis of a current carrying wire is always zero.

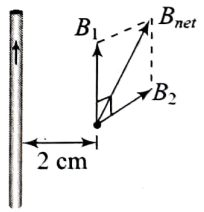


6. (d) Use right-hand palm rule, or Maxwell's cork screw rule or any other.

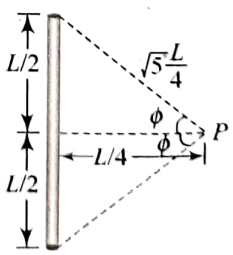
7. (c) $B_1 = 4\times10^{-4}$ T

$$B_2 = 10^{-7}\times\frac{2\times30}{2\times10^{-2}} = 3\times10^{-4}\text{T}$$

$$\therefore \quad B_{net} = \sqrt{B_1^2 + B_2^2} = 5\times10^{-4}\text{T}$$



8. (a) By using $B = \frac{\mu_0}{4\pi}\cdot\frac{i}{a}(\sin\phi_1 + \sin\phi_2)$



$$\Rightarrow \quad B = \frac{\mu_0}{4\pi}\cdot\frac{i}{(L/4)}(2\sin\phi)$$

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Also $\sin\phi = \frac{L/2}{\sqrt{5}L/4} = \frac{2}{\sqrt{5}}$

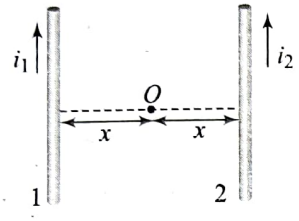
$\Rightarrow \quad B = \frac{4\mu_0 i}{\sqrt{5}\pi L}$

9. (d) Initially when wires carry currents in the same direction as shown.

Magnetic field at mid-point O due to wires 1 and 2 are respectively

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2i_1}{x}\otimes$$

and $$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2i_2}{x}\odot$$



Hence net magnetic field at O $B_{net} = \frac{\mu_0}{4\pi}\times\frac{2}{x}\times(i_1 - i_2)$

$\Rightarrow \quad 10\times10^{-6} = \frac{\mu_0}{4\pi}\cdot\frac{2}{x}(i_1 - i_2)$ (i)

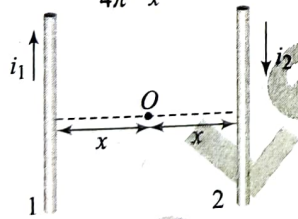
If the direction of i_2 is reversed, then

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2i_1}{x}\otimes$$

and $$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2i_2}{x}\otimes$$

So $$B_{net} = \frac{\mu_0}{4\pi}\cdot\frac{2}{x}(i_1 + i_2)$$

$\Rightarrow \quad 40\times10^{-6} = \frac{\mu_0}{4\pi}\cdot\frac{2}{x}(i_1 + i_2)$ (ii)



Dividing equation (ii) by (i) $\frac{i_1 + i_2}{i_1 - i_2} = \frac{4}{1} \Rightarrow \frac{i_1}{i_2} = \frac{5}{3}$

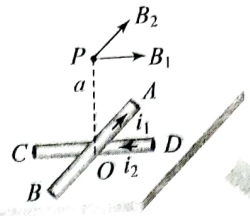
$$B_X = \frac{\mu_0}{4\pi}\cdot\frac{2\pi i r^2}{\left\{\left(\frac{d}{2}\right)^2 + r^2\right\}^{3/2}} = \frac{\mu_0}{4\pi}\cdot\frac{16\pi i r^2}{(d^2 + 4r^2)^{3/2}}$$

$\Rightarrow \quad \frac{B_Y}{B_X} = \frac{1}{2}$

10. (c) At P, $B_{net} = \sqrt{B_1^2 + B_2^2}$

$$= \sqrt{\left(\frac{\mu_0}{4\pi}\frac{2i_1}{a}\right)^2 + \left(\frac{\mu_0}{4\pi}\frac{2i_2}{a}\right)^2}$$

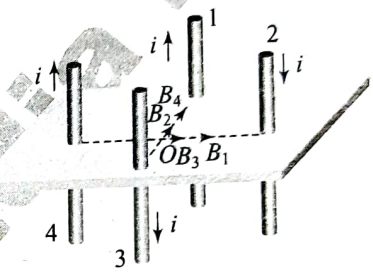
$$= \frac{\mu_0}{2\pi a}(i_1^2 + i_2^2)^{1/2}$$



11. (c) Direction of magnetic field (B_1, B_2, B_3 and B_4) at origin due to wires 1, 2, 3 and 4 are shown in the following figure.

$B_1 = B_2 = B_3 = B_4 = \frac{\mu_0}{4\pi}\cdot\frac{2i}{x} = B$. So net magnetic field at origin O

$$B_{net} = \sqrt{(B_1 + B_3)^2 + (B_2 + B_4)^2} = \sqrt{(2B)^2 + (2B)^2} = 2\sqrt{2}B$$



12. (b) The field at the midpoint of BC due to AB is $\left(-\frac{\mu_0}{4\pi}\cdot\frac{I}{d/2}\hat{k}\right)$ and the same is due to CD. Therefore the total field is $\left[-\left(\frac{\mu_0 I}{\pi d}\right)\hat{k}\right]$

13. (b) According to question resistance of wire ADC is twice that of wire ABC. Hence current flows through ADC is half that of ABC i.e. $\frac{i_2}{i_1} = \frac{1}{2}$. Also $i_1 + i_2 = i \Rightarrow i_1 = \frac{2i}{3}$ and $i_2 = \frac{i}{3}$

Magnetic field at centre O due to wire AB and BC (parts 1 and 2)

$$B_1 = B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2i_1\sin 45^\circ}{a/2}\otimes = \frac{\mu_0}{4\pi}\cdot\frac{2\sqrt{2}i_1}{a}\otimes$$

and magnetic field at centre O due to wires AD and DC (i.e. parts 3 and 4) $B_3 = B_4 = \frac{\mu_0}{4\pi}\frac{2\sqrt{2}i}{a}\odot$

Also $i_1 = 2i_2$. So $(B_1 = B_2) > (B_3 = B_4)$

Hence net magnetic field at centre O

$$B_{net} = (B_1 + B_2) - (B_3 + B_4)$$

$$= 2\times\frac{\mu_0}{4\pi}\cdot\frac{2\sqrt{2}\times\left(\frac{2}{3}i\right)}{a} - \frac{\mu_0}{4\pi}\cdot\frac{2\sqrt{2}\left(\frac{i}{3}\right)\times 2}{a}$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{4\sqrt{2}i}{3a}(2-1)\otimes = \frac{\sqrt{2}\mu_0 i}{3\pi a}\otimes$$

14. (d) In P and R loops, currents are divided in same proportion because the branches have equal resistance. Hence magnetic field produced at centre due to each segment is of equal magnitude but of opposite direction, so net field is zero.

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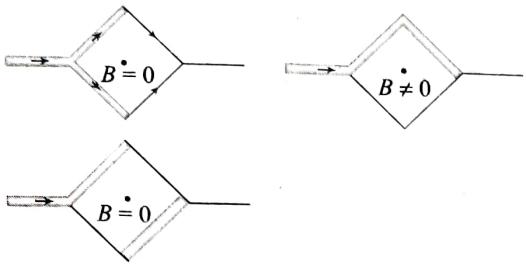
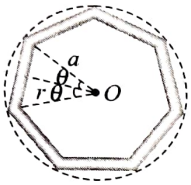


15. (b) Magnetic field at the centre due to one side

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2i\sin\theta}{r} \quad \text{where } r = a\cos\theta$$

So $$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2i\sin\theta}{a\cos\theta} = \frac{\mu_0 i}{2\pi a}\tan\theta$$



Hence net magnetic field

$$B_{net} = n\times\frac{\mu_0 i}{2\pi a}\tan\frac{\pi}{n}$$

DPP 2.2

Subjective Type

1. (a) The radial segments do not contribute to $\vec{B}_p$. If $\hat{k}$ designates the direction "out of the page, then

$$\vec{B} = \frac{\mu_0(0.40\text{ A})(\pi\text{ rad})}{4\pi(0.050\text{ m})}\hat{k} - \frac{\mu_0(0.80\text{ A})(2\pi/3\text{ rad})}{4\pi(0.040\text{ m})}\hat{k}$$

$$= -(1.7\times10^{-6}T)\hat{k}$$

or $$|\vec{B}| = 1.7\times10^{-6}T$$

(b) The direction is $-\hat{k}$ or into the page.

(c) If the direction of i_1 is reversed, we then have

$$\vec{B} = -\frac{\mu_0(0.40\text{ A})(\pi\text{ rad})}{4\pi(0.05\text{ m})}\hat{k} - \frac{\mu_0(0.80\text{ A})(2\pi/3\text{ rad})}{4\pi(0.040\text{ m})}\hat{k}$$

$$= -(6.7\times10^{-6}T)\hat{k}$$

or $$|\vec{B}| = 6.7\times10^{-6}T$$

(d) The direction is $-\hat{k}$, or into the page.

2. In the one case we have $B_{small} + B_{big} = 47.25\ \mu T$, and the other case gives $B_{small} - B_{big} = 15.75\ \mu T$ (cautionary note about our notation: B_{small} refers to the field at the center of the small-radius arc, which is actually a bigger field than B_{big}!). Dividing one of these equations by the other and canceling out common factors we obtain

$$\frac{(1/r_{small}) + (1/r_{big})}{(1/r_{small}) - (1/r_{big})} = \frac{1 + (r_{small}/r_{big})}{1 - (r_{small}/r_{big})} = 3$$

The solution of this is straightforward: $r_{small} = r_{big}/2$. Using the given fact that the $r_{big} = 4.00$ cm. then we conclude that the small radius is $r_{small} = 2.00$ cm.

3. We refer to the center of the circle (where we are evaluating $\vec{B}$) as C. We see that the current in the straight segments that are collinear with C do not contribute to the field there. Let us now calculate net magnetic field due to circular arcs

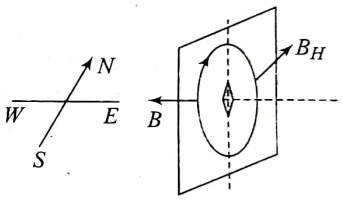
$$B_{circular} = \frac{\mu_0 i(\pi/2)}{4\pi R} - \frac{\mu_0 i(\pi/2)}{4\pi R} = 0$$

to the field at C. Thus, the nonzero contributions come from those straight segments that are not collinear with C. There are two of these "semi-infinite" segments, one a vertical distance R above C and the other a horizontal distance R to the left of C. Both contribute fields pointing out of the page. Since the magnitudes of the two contributions add, then the result is

$$B = 2\left(\frac{\mu_0 i}{4\pi R}\right) = \frac{\mu_0 i}{2\pi R}$$

Single Correct Answer Type

4. (b)



5. (a) Magnetic field due to revolution of electron

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi i}{r} = \frac{\mu_0}{4\pi}\cdot\frac{2\pi\cdot\left(\frac{e\omega}{2\pi}\right)}{r} = 10^{-7}\times\frac{e\omega}{r}$$

$$\Rightarrow \quad 16 = 10^{-7}\times\frac{1.6\times10^{-19}\omega}{1\times10^{-10}} \Rightarrow \omega = 10^{17}\text{ rad/sec}$$

6. (b, c)

Since length of the wire is equal to l, therefore, $2\pi Rn = l$ or $n = \frac{l}{2\pi R}$.

Magnetic induction at centre of a circular coil is given by

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi ni}{R} = \frac{\mu_0 l i}{4\pi R^2} \Rightarrow B \propto \frac{1}{R^2}$$

It means, when $R \to 0$, $B \to \infty$ and $R \to \infty$, $B \to 0$.

Hence (b) is correct and (d) is wrong.

Substituting $R = \frac{l}{2\pi n}$ in $B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi ni}{R}$

$B \propto n^2$. It means graph between B and n will be a parabola having increasing slope and passing through origin. Hence (c) is correct and (a) is wrong.

7. (d) Magnetic meridian is a vertical N-S plane, the earth's magnetic field (B_H) lies in it.

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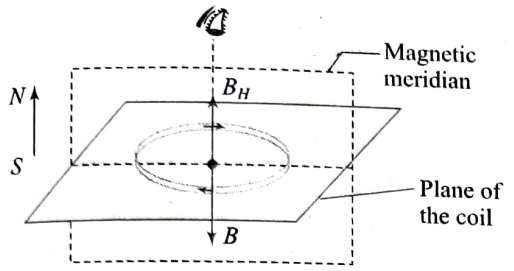
To obtain neutral point at the centre of coil, magnetic field due to current (B) and B_H must cancel each other. Hence plane of the coil and magnetic meridian must be perpendicular to each other as shown



8. (d) Magnetic field at centre due to smaller loop

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2\pi i_1}{r_1} \qquad \text{(i)}$$

Due to bigger loop $B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2\pi i_2}{r_2}$ So net magnetic field at centre

$$B = B_1 - B_2 = \frac{\mu_0}{4\pi}\times 2\pi\left(\frac{i_1}{r_1}-\frac{i_2}{r_2}\right)$$

According to question $B = \frac{1}{2}\times B_1$

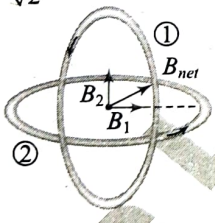
$$\frac{\mu_0}{4\pi}\cdot 2\pi\left(\frac{i_1}{r_1}-\frac{i_2}{r_2}\right) = \frac{1}{2}\times\frac{\mu_0}{4\pi}\cdot\frac{2\pi i_1}{r_1}$$

$$\Rightarrow \quad \frac{i_1}{r_1}-\frac{i_2}{r_2} = \frac{i_1}{2r_1} \Rightarrow \frac{i_1}{2r_1} = \frac{i_2}{r_2} \Rightarrow \frac{i_1}{i_2} = 1 \qquad \{r_2 = 2r_1\}$$

9. (a) $B_1 = B_2 = B = \frac{\mu_0}{4\pi}\times\frac{2\pi i}{r}$

$B_{net} = \sqrt{2}B$

$$\Rightarrow \quad \frac{B}{B_{net}} = \frac{1}{\sqrt{2}}$$



10. (d) $B = \frac{\mu_0}{4\pi}\frac{(2\pi-\theta)i}{R} = \frac{\mu_0}{4\pi}\frac{\left(2\pi-\frac{\pi}{2}\right)\times i}{R} = \frac{3\mu_0 i}{8R}$

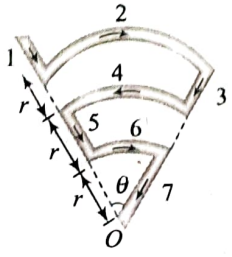
11. (c) The magnetic induction due to both semicircular parts will be in the same direction perpendicular to the paper inwards.

$$\therefore \quad B = B_1 + B_2 = \frac{\mu_0 i}{4r_1} + \frac{\mu_0 i}{4r_2} = \frac{\mu_0 i}{4}\left(\frac{r_1+r_2}{r_1 r_2}\right)\otimes$$

12. (a) $B_1 = B_2 = B_5 = 0$

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{\theta i}{3r}\otimes,\ B_4 = \frac{\mu_0}{4\pi}\cdot\frac{\theta i}{2r}\odot$$

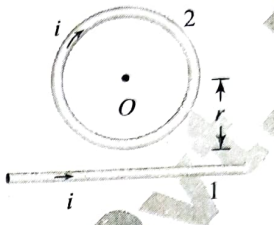
and $B_6 = \frac{\mu_0}{4\pi}\cdot\frac{\theta i}{r}\otimes$



$\therefore$ Net magnetic field at O,

$$B_{net} = B_2 - B_4 + B_6 = \frac{\mu_0}{4\pi}\cdot\frac{\theta i}{r}\left(\frac{1}{3}-\frac{1}{2}+1\right) = \frac{5\mu_0\theta i}{24\pi r}$$

13. (b) The given shape is equivalent to the following diagram

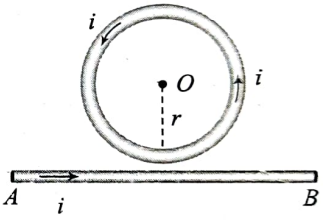


The field at O due to straight part of conductor is $B_1 = \frac{\mu_o}{4\pi}\cdot\frac{2i}{r}\odot$

The field at O due to circular coil is $B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2\pi i}{r}\otimes$. Both fields will act in the opposite direction, hence the total field at O.

$$\Rightarrow \quad B = B_2 - B_1 = \left(\frac{\mu_o}{4\pi}\right)\times(\pi-1)\frac{2i}{r} = \frac{\mu_o}{4\pi}\cdot\frac{2i}{r}(\pi-1)$$

14. (c) The given circuit can be considered as follows

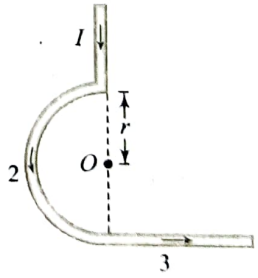


$$B_{loop} = \frac{\mu_0 i}{2r}\odot$$

$$B_{conductor} = \frac{\mu_0 i}{2\pi r}\odot$$

$$B_{net} = \frac{\mu_0 i}{2\pi r}(\pi+1)\odot$$

15. (c) Magnetic field due to different parts are



$B_1 = 0$

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{\pi I}{r}\odot$$

$$B_3 = \frac{\mu_0}{4\pi}\cdot\frac{I}{r}\odot$$

$$\therefore \quad B_{net} = B_2 + B_3 = \frac{\mu_0 I}{4r} + \frac{\mu_0 I}{4\pi r}$$

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DPP 2.3

Subjective Type

1. Letting "out of the page" in Fig. (a) be the positive direction, the net field is

$$B = \frac{\mu_0 i_1 \phi}{4\pi R} - \frac{\mu_0 i_2}{2\pi(R/2)}$$

In Figure (b) (see question), we see that $B = 0$ when $i_2 = 0.5$ A, so (solving the above expression with B set equal to zero) we must have

$$\phi = 4(i_2/i_1) = 4(0.5/2) = 1.00 \text{ rad}$$

2. Initially we have $B_i = \frac{\mu_0 i\phi}{4\pi R} + \frac{\mu_0 i\phi}{4\pi r}$

In the final situation, we use Pythagorean theorem and write

$$B_f^2 = B_z^2 + B_y^2 = \left(\frac{\mu_0 i\phi}{4\pi R}\right)^2 + \left(\frac{\mu_0 i\phi}{4\pi r}\right)^2$$

If we square B_i and divide by B_y^2, we obtain

$$\left(\frac{B_i}{B_f}\right) = \frac{[(1/R) + (1/r)]^2}{(1/R)^2 + (1/r)^2}$$

From the graph (see Fig. (c) given in the question), note the maximum and minimum values) we estimate $B_i/B_f = 12/10 = 1.2$, and this allows us to solve for r in terms of R:

$$r = R\frac{1 \pm 1.2\sqrt{2 - 1.2^2}}{1.2^2 - 1} = 2.3 \text{ cm} \quad \text{or} \quad 43.1 \text{ cm}$$

Since we require $r < R$, then the acceptable answer is $r = 2.3$ cm.

Single Correct Answer Type

3. (b) Direction of magnetic field at every point on axis of a current carrying coil remains same though magnitude varies. Hence magnetic induction for whole the x-axis will remain positive. Therefore, (c) and (d) are wrong.

Magnitude of magnetic field will vary with x according to law,

$$B = \frac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}}.$$

Hence, at $x = 0$, $B = \frac{\mu_0 NI}{2R}$ and when $x \to \infty$, $B \to 0$

Slope of the graph will be $\frac{dB}{dx} = -\frac{3\mu_0 NIR^2 \cdot x}{2(R^2 + x^2)^{5/2}}$

It means, at $x = 0$, slope is equal to zero or tangent to the graph at $x = 0$, must be parallel to x-axis.

Hence (b) is correct and (a) is wrong.

4. (b) $\frac{B_{\text{centre}}}{B_{\text{axis}}} = \left(1 + \frac{x^2}{R^2}\right)^{3/2}$, also $B_{\text{axis}} = \frac{1}{8} B_{\text{centre}}$

$$\Rightarrow \quad \frac{8}{1} = \left(1 + \frac{x^2}{R^2}\right)^{3/2} \Rightarrow 2 = \left(1 + \frac{x^2}{R^2}\right)^{1/2}$$

$$\Rightarrow \quad 4 = 1 + \frac{x^2}{R^2} \Rightarrow 3 = \frac{x^2}{R^2} \Rightarrow x^2 = 3R^2 \Rightarrow x = \sqrt{3}R$$

5. (c) Magnetic field at O due to bigger coil Y is

$$B_Y = \frac{\mu_0}{4\pi} \cdot \frac{2\pi i(2r)^2}{\{d^2 + (2r)^2\}^{3/2}} = \frac{\mu_0}{4\pi} \cdot \frac{8\pi i r^2}{(d^2 + 4r^2)^{3/2}}$$

Magnetic field at O due to smaller coil X is

$$B_X = \frac{\mu_0}{4\pi} \cdot \frac{2\pi i(r)^2}{\{(d/2)^2 + r^2\}^{3/2}} = \frac{\mu_0}{4\pi} \cdot \frac{16\pi i r^2}{(d^2 + 4r^2)^{3/2}}$$

Hence, $\frac{B_Y}{B_X} = \frac{1}{2}$

6. (a) Field at the centre $B_1 = \frac{\mu_0}{4\pi} \times \frac{2\pi i n}{r} = \frac{\mu_0}{2} \cdot \frac{ni}{r}$

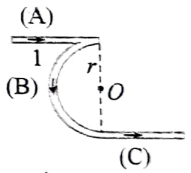
Field at a distance h from the centre

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n i r^2}{(r^2 + h^2)^{3/2}} = \frac{\mu_0}{2} \cdot \frac{nir^2}{r^3\left(1 + \frac{h^2}{r^2}\right)^{3/2}}$$

$$= B_1\left(1 + \frac{h^2}{r^2}\right)^{-3/2} = B_1\left(1 - \frac{3}{2} \cdot \frac{h^2}{r^2}\right) \text{ (By binomial theorem)}$$

Hence B_2 is less than B_1 by a fraction $= \frac{3}{2}\frac{h^2}{r^2}$

7. (a) **Case 1:**



$$B_A = \frac{\mu_0}{4\pi} \cdot \frac{i}{r} \otimes$$

$$B_B = \frac{\mu_0}{4\pi} \cdot \frac{\pi i}{r} \odot$$

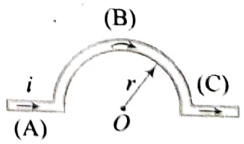
$$B_C = \frac{\mu_0}{4\pi} \cdot \frac{i}{r} \odot$$

So net magnetic field at the centre of case 1

$$B_1 = B_B - B_C - B_A \Rightarrow B_1 = \frac{\mu_0}{4\pi} \cdot \frac{\pi i}{r} \odot \quad \text{(i)}$$

Case 2: As we discussed before magnetic field at the centre O in this case

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{\pi i}{r} \otimes \quad \text{(ii)}$$

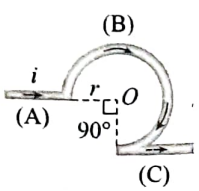


Case 3: $B_A = 0$

$$B_B = \frac{\mu_0}{4\pi} \cdot \frac{(2\pi - \pi/2)i}{r} \otimes$$

$$B_C = \frac{\mu_0}{4\pi} \cdot \frac{i}{r} \odot$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{3\pi i}{2r} \otimes$$



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So net magnetic field at the centre of case 3

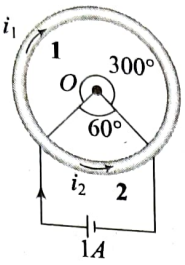
$$B_3 = \frac{\mu_0}{4\pi}\cdot\frac{i}{r}\left(\frac{3\pi}{2}-1\right)\otimes \qquad \text{(iii)}$$

From equation (i), (ii) and (iii)

$$B_1 : B_2 : B_3 = \pi\odot : \pi\otimes$$

$$\left(\frac{3\pi}{2}-1\right)\otimes = -\frac{\pi}{2} : \frac{\pi}{2} : \left(\frac{3\pi}{4}-\frac{1}{2}\right)$$

8. (c) $B = \frac{\mu_0}{4\pi}\frac{\theta i}{r} \Rightarrow B \propto \theta i \quad \left(\text{but } \frac{i_1}{i_2} = \frac{l_2}{l_1} = \frac{\theta_2}{\theta_1}\right)$

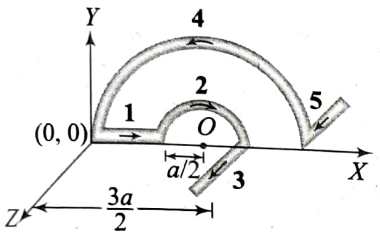


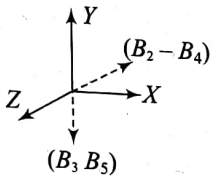
$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{\theta_1}{\theta_2}\cdot\frac{i_1}{i_2}$$

So, $\quad \frac{B_1}{B_2} = \frac{\theta_1}{\theta_2}\times\frac{\theta_2}{\theta_1}$

$$\Rightarrow \quad B_1 = B_2$$

9. (b)





Magnetic field at 0 due to

Part (1): $B_1 = 0$

Part (2): $B_2 = \frac{\mu_0}{4\pi}\cdot\frac{\pi i}{(a/2)}\otimes$ (along –Z-axis)

Part (3): $B_3 = \frac{\mu_0}{4\pi}\cdot\frac{i}{(a/2)}(\downarrow)$ (along –Y-axis)

Part (4): $B_4 = \frac{\mu_0}{4\pi}\frac{\pi i}{(3a/2)}\odot$ (along +Z-axis)

Part (5): $B_5 = \frac{\mu_0}{4\pi}\cdot\frac{i}{(3a/2)}(\downarrow)$ (along –Y-axis)

$$B_2 - B_4 = \frac{\mu_0}{4\pi}\cdot\frac{\pi i}{a}\left(2-\frac{2}{3}\right) = \frac{\mu_0 i}{3a}\otimes \quad \text{(along –Z-axis)}$$

$$B_3 + B_5 = \frac{\mu_0}{4\pi}\cdot\frac{1}{a}\left(2+\frac{2}{3}\right) = \frac{8\mu_0 i}{12\pi a}(\downarrow) \quad \text{(along –Y-axis)}$$

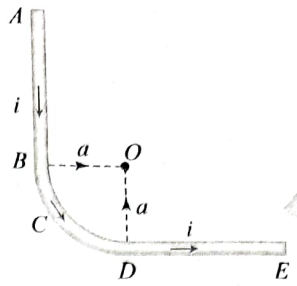
Hence net magnetic field

$$B_{net} = \sqrt{(B_2 - B_4)^2 + (B_3 + B_5)^2} = \frac{\mu_0 i}{3\pi a}\sqrt{\pi^2 + 4}$$

10. (d) The field at 0 due to AB is $\frac{\mu_0}{4\pi}\cdot\frac{i}{a}\hat{k}$ and that due to DE is also $\frac{\mu_0}{4\pi}\cdot\frac{i}{a}\hat{k}$.

However the field due to BCD is $\frac{\mu_0}{4\pi}\cdot\frac{i}{a}\left(\frac{\pi}{2}\right)\hat{k}$.

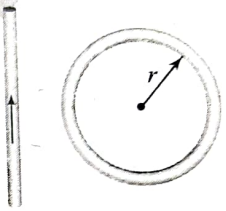
Thus the total field at O is $\frac{\mu_0}{4\pi}\cdot\frac{i}{a}\left(2+\frac{\pi}{2}\right)\hat{k}$.



11. (b) If a wire of length l is bent in the form of a circle of radius r then $2\pi r \Rightarrow l$

$$r = \frac{l}{2\pi}$$

$$\Rightarrow \quad r = \frac{l}{2\pi} = \frac{\pi^2}{2\pi} = \frac{\pi}{2}$$



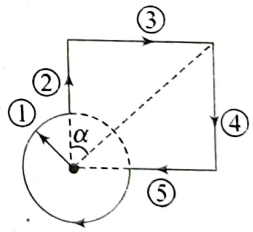
Magnetic field due to straight wire $B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2i}{r} = \frac{\mu_0}{4\pi}\times\frac{2\times 2}{1\times 10^{-2}}$

also magnetic field due to circular loop

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2\pi i}{r} = \frac{\mu_0}{4\pi}\cdot\frac{2\pi\times 2}{\pi/2}$$

$$\Rightarrow \quad \frac{B_2}{B_1} = \frac{1}{50}$$

12. (b) Magnetic field due to wire reactions (2) and (3) will be zero.



Magnetic field due to straight parts (3) and (4) will be equal

$$\vec{B}_3 = \vec{B}_4 = \frac{\mu_0 i}{4\pi d}[\sin\theta + \sin\alpha]\otimes$$

$$= \frac{\mu_0 i}{4\pi d}[\sin 45^\circ] = \frac{\mu_0 i}{4\sqrt{2}\,\pi d}\otimes$$

Magnetic field due to circular arc (1)

$$\vec{B}_1 = \left(\frac{\mu_0 i}{2R}\right)\cdot\frac{3}{4}\otimes$$

$$\vec{B}_{net} = \vec{B}_1 + 2\vec{B}_3 = \frac{\mu_0 i}{4\pi}\left[\frac{3\pi}{2R} + \frac{\sqrt{2}}{d}\right]\otimes$$

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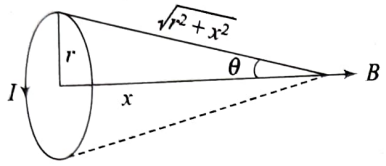
Multiple Correct Answers Type

13. (a, b)

The magnetic field due to straight wires will be zero as both are directed towards the centre of the circle. The magnetic field due to circular arc will be cancelled out. Hence magnetic field due to loop will be zero.

14. (b, d)

$$B=\frac{\mu_0 N I r^2}{2(r^2+x^2)^{3/2}} \text{ along } x\text{-axis}$$



$$B=\frac{\mu_0 NI}{2r}\frac{r^3}{(r^2+x^2)^{3/2}}$$

$$B=\frac{\mu_0 NI}{2r}\sin^3\theta$$

15. (a, b, d)

Magnetic field due to wire 1

- At position P and R is zero
- At position Q and S are equal and opposite $\frac{\mu_0 I}{4d}$, $\odot$ at S and $\otimes$ at Q.

Magnetic field due to wire 2

- At positions S and Q is zero
- At position P and R is $\frac{\mu_0 I}{4d}$ $\odot$ at P and $\otimes$ at R.
- Hence net magnetic field at points P, Q, R and S are same in magnitude.
- Direction of magnetic field at Q & R and S & P are same.
- Direction of magnetic field at R & S and P & Q are opposite.

DPP 2.4

Subjective Type

1. Current in the element $= J(2\pi r.dr)$

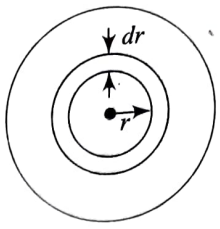
Current enclosed by amperian loop of radius $\frac{a}{2}$

$$I=\int_0^{a/2}\frac{J_0 r}{a}\cdot 2\pi r\cdot dr=\frac{2\pi J_0}{3a}\left(\frac{a}{2}\right)^3=\frac{\pi J_0 a^2}{12}$$

Applying Ampere's law

$$B\cdot 2\pi\cdot\frac{a}{2}=\mu_0\cdot\frac{\pi J_0 a^2}{12}\Rightarrow B=\frac{\mu_0 J_0 a}{12}$$

On putting values $B = 10\ \mu T$



2. We use Ampere's law: $\oint \vec{B}\cdot d\vec{s}=\mu_0 i$, where the integral is around a closed loop and i is the net current through the loop.

(a) For path 1, the result is

$$\oint_1 \vec{B}\cdot d\vec{s}=\mu_0(-5.0\text{ A}+3.0\text{ A})$$

$$=(4\pi\times10^{-7}\text{ T.m/A})(-2.0\text{ A})=-2.5\times10^{-6}\text{ T.m}$$

(b) For path 2, we find

$$\oint_2 \vec{B}\cdot d\vec{s}=\mu_0(-5.0\text{ A}-5.0\text{ A}-3.0\text{ A})$$

$$=(4\pi\times10^{-7}\text{ T.m/A})(-13.0\text{ A})=-1.6\times10^{-5}\text{ T.m}$$

3. The value of the line integral $\oint \vec{B}\cdot d\vec{s}$ is proportional to the net current enclosed.

By Ampere's law, we have $\oint \vec{B}\cdot d\vec{s}=\mu_0 i_{enc}$, where i_{enc} is the current enclosed by the closed path.

(a) Two of the currents are out of the page and one is into the page, so the net current enclosed by the path, or "amperian loop" is 2.0 A, out of the page. Since the path is traversed in the clockwise sense, a current into the page is positive and a current out of the page is negative, as indicated by the right-hand rule associated with Ampere's law. Thus,

$$\oint \vec{B}\cdot d\vec{s}=\mu_0 i=-(4\pi\times10^{-7}\text{ T.m/A})(2.0\text{ A})$$

$$=-2.5\times10^{-6}\text{ T.m}$$

(b) The net current enclosed by the path is zero (two currents are out of the page and two are into the page), so $\oint \vec{B}\cdot d\vec{s}=\mu_0 i_{enc}=0$.

The value of $\oint \vec{B}\cdot d\vec{s}$ depends only on the current enclosed, and not the shape of the amperian loop.

Single Correct Answer Type

4. (c) For calculation of $\oint_{ABCDA} \vec{B}\cdot d\vec{l}$, current upwards shall be positive and current downwards shall be negative.

Therefore $\oint_{ABCDA} \vec{B}\cdot d\vec{l}=(I_1-I_1)+I_2-4I_1=0$

Therefore $I_2=4I_1$ upwards.

5. (b) We shall use $\oint \vec{B}\cdot d\vec{l}=\mu_0 I$ where I is the current enclosed by loop.

Net current enclosed by path a is zero.
Net current enclosed by path c is A.
Net current enclosed by path d is $3A$.
Net current enclosed by path b is $5A$.

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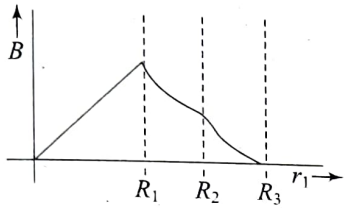
6. (a) A close look at the path reveals that only currents 1, 3, 6 and 7 are enclosed. Thus, noting the different current directions described in the problem, we obtain

$$\oint \vec{B}.d\vec{s} = \mu_0 (7i - 6i + 3i + i) = 5\mu_0 i = 10\mu_0$$

7. (a) Magnetic field inside the hollow metallic cylinder $B_{in} = 0$, and magnetic field outside it $B_{out} \propto \frac{1}{r}$.

8. (c) From Ampere's law, the field at the axis is zero.
From x = zero to R_1, the field increases linearly as the charge enclosed increases.
From $X = R_1$ to R_2 and from $X = R_2$ to R_3, the field decreases hyperbolically but with different slopes as the media are different.
Hence the required graph is

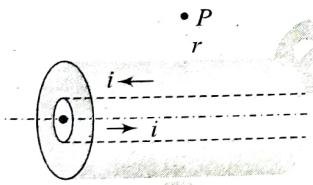


9. (d) By using $B = \frac{\mu_0 i}{2\pi r}\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$ here

$$r = \frac{3R}{2},\ a = R,\ b = 2R$$

$$\Rightarrow \quad B = \frac{\mu_0 i}{2\pi\left(\frac{3R}{2}\right)} \times \left\{\frac{\left(\frac{3R}{2}\right)^2 - R^2}{(2R^2) - R^2}\right\} = \frac{5 \cdot \mu_o i}{36\pi r}$$

10. (b) The respective figure is shown below
Magnetic field at P due to inner and outer conductors are equal and opposite. Hence net magnetic field at P will be zero.



11. (d) For cylinder;

$$B = \frac{\mu_0 i r}{2\pi R^2};\quad r < a$$

$$= \frac{\mu_0 i}{2\pi r};\quad r \geq a$$

We can consider the given cylinder as a combination of two cylinders. One of radius 'R' carrying current I in one direction and other of radius $\frac{R}{2}$ carrying current $\frac{I}{3}$ in both directions.

At point A: $B = \frac{\mu_0 (I/3)}{2\pi (R/2)} + 0 = \frac{\mu_0 I}{3\pi R}$

At point B: $B = \frac{\mu_0}{2}\left(\frac{4I/3}{\pi R^2}\right)\left(\frac{R}{2}\right) + 0 = \frac{\mu_0 I}{3\pi R}$

12. (c) $B = \frac{\mu_0 N i}{l}$ where N = Total number of turns, l = length of the solenoid

$$\Rightarrow \quad 0.2 = \frac{4\pi \times 10^{-7} \times N \times 10}{0.8} \Rightarrow N = \frac{4 \times 10^4}{\pi}$$

Since N turns are made from the winding wire so length of the wire (L) = $2\pi r \times N$ [$2\pi r$ length of each turns]

$$\Rightarrow \quad L = 2\pi \times 3 \times 10^{-2} \times \frac{4 \times 10^4}{\pi} = 2.4 \times 10^3 \text{ m}$$

13. (b) Magnetic field inside the cylindrical conductor

$B_{in} = \frac{\mu_0}{4\pi} \cdot \frac{2ir}{R^2}$ (R = Radius of cylinder, r = distance of observation point from axis of cylinder)

Magnetic field outside the cylinder at a distance r' from its axis

$$B_{out} = \frac{\mu_0}{4\pi} \cdot \frac{2i}{r'}$$

$$\Rightarrow \quad \frac{B_{in}}{B_{out}} = \frac{rr'}{R^2} \Rightarrow \frac{10}{B_{out}} = \frac{\left(R - \frac{R}{4}\right)(R + 4R)}{R^2} \Rightarrow B_{out} = \frac{8}{3}\text{T}$$

Multiple Correct Answers Type

14. (c, d)

$$B_{net} = B_1 - B_2 \Rightarrow B_1 - B_2 = 0 \Rightarrow B_1 = B_2$$

$$\Rightarrow \quad B \propto ni. \text{ So } n_1 i_2 = n_2 i_2 \text{ or } n_1 = n_2 \text{ and } i_1 = i_2$$

15. (b, c)
Applying the Ampere's circuital law, we have

$\oint B \cdot dl = \mu_0 (I - I) = 0$ (because current is in opposite sense.)

Also, there may be a point on C where B and dl are perpendicular and hence, $\oint_c B \cdot dl = 0$.

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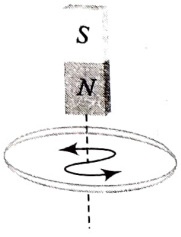
CHAPTER 3

DPP 3.1

Single Correct Answer Type

1. (c) As it is seen from the magnet side induced current will be anticlockwise



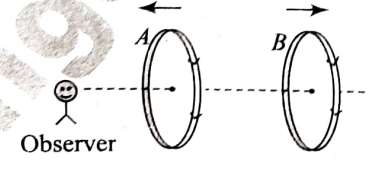
2. (b) When the magnet is allowed to fall vertically along the axis of loop with its north pole towards the ring. The upper face of the ring will become north pole in an attempt to oppose the approaching north pole of the magnet. Therefore the acceleration in the magnet is less than g.

3. (a) If coil is broken at any point, then induced *emf* will be generated in it but no induced current will flow. In this condition the coil will not oppose the motion of magnet and the magnet will fall freely with acceleration g (i.e. $a = g$).

4. (a) If bar magnet is falling vertically through the hollow region of long vertical copper tube, then the magnetic flux linked with the copper tube (due to 'non-uniform' magnetic field of magnet) changes and eddy currents are generated in the body of the tube by Lenz's law. The eddy currents opposes the falling of the magnet which therefore experience a retarding force. The retarding force increases with increasing velocity of the magnet and finally equals the weight of the magnet. The magnet then attains a constant final terminal velocity *i.e.* magnet ultimately falls with zero acceleration in the tube.

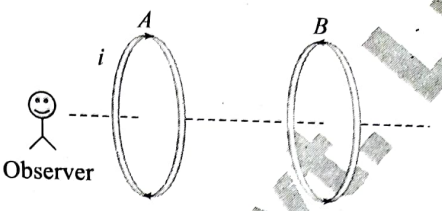
5. (d) If electron is moving from left to right, the flux linked with the loop (which is into the page) will first increase and then decrease as the electron passes by. So the induced current in the loop will be first anticlockwise and will change direction as the electron passes by.

6. (a) Induced current in both the coils assist the main current so current through each coil increases.



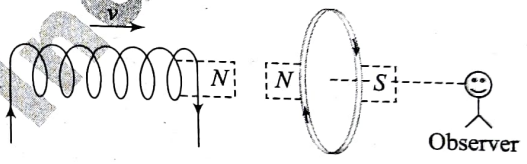
7. (d) Emf is induced in the ring, and it will oppose the motion. Hence due to the resistance of the ring, all energy dissipates.

8. (d)



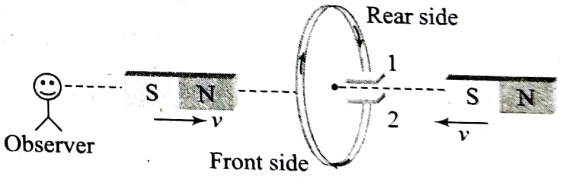
If current through A increases, crosses (X) linked with coil B increases, hence anticlockwise current induces in coil B. As shown in the figure both the current produces repulsive effect.

9. (b) The direction of current in the solenoid is anticlockwise as seen by observer. On displacing it towards the loop, a current in the loop will be induced in a direction so as to oppose the approach of solenoid. Therefore the direction of induced current as observed by the observer will be clockwise.



10. (c) When key k is pressed, current through the electromagnet start increasing *i.e.* flux linked with ring increases which produces repulsion effect.

11. (b) By the movement of both the magnets, current will be anticlockwise, as seen from left side *i.e.* plate 1 will be positive and plate 2 will be negative.



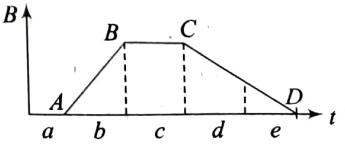
12. (d) At B, flux is maximum, so from $|e| = \frac{d\phi}{dt}$ at B $|e| = 0$

13. (d) When loop enters in field between the pole pieces, flux linked with the coil first increases (constantly) so a constant emf induces, when coil is entered completely within the field, no flux change so $e = 0$.
When coil exits out, flux linked with the coil decreases, hence again emf induces, but in opposite direction.

14. (a) With rise in current in coil A flux through B increases. According to Lenz's law repulsion occurs between A and B.

15. (b) Induced emf $e = A\frac{dB}{dt}$

i.e. $e \propto \frac{dB}{dt}$ (= slope of B–t graph)



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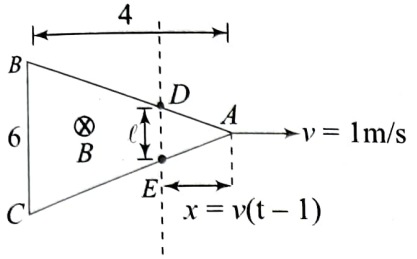
In the given graph slope of AB > slope of CD, slope in the 'a' region = slope in the 'c' region = 0, slope in the 'd' region = slope in the 'e' region ≠ 0. That's why $b > (d = e) > (a = c)$.

DPP 3.2

Subjective Type

1. (i) $\varepsilon = 00 \le t < 1$ (Since, change in flux is zero)



Now let at any later time the x length of triangle be outside the magnetic field, then

(ii) By similar triangles, ABC and ADE,

$$DE = l = \frac{3x}{2} = \frac{2}{3}v(t-1)$$

Area of the triangle $ADE = \frac{1}{2}xl = \frac{1}{2}x\left(\frac{3x}{2}\right) = \frac{2x^2}{4}$

Area of trapezium inside the magnetic field

$$= 12 - \frac{3x^2}{4}$$

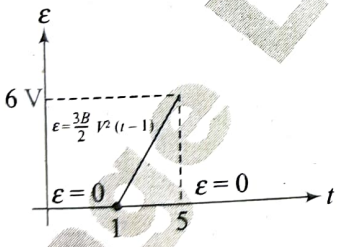
so induced emf in the loop is

$$\Rightarrow \quad \varepsilon = \frac{d(BA)}{dt} = \frac{3Bx}{2}\frac{dx}{dt} = \frac{3Bv^2}{2}(t-1)$$

$$\varepsilon = \frac{3}{2}BV^2(t-1)\ (t \le t \le)$$

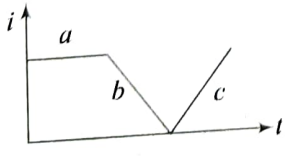
(iii) $\varepsilon = 0 \quad t > 5$

(since magnetic field is zero)



Single Correct Answer Type

2. (c) Emf induces during 'a' = 0
emf induced during 'b' is constant throughout emf induced during 'c' is constant throughout magnitude of emf induced during 'b' is equal to the magnitude of emf induced during 'c'. But the direction is opposite.



3. (b) As x increases, $\frac{dB}{dt}$ increases, i.e., induced emf (E) is negative.

When loop is completely entered in the magnetic field, emf = 0

When it exits out x increases but $\frac{dB}{dt}$ decreases, i.e., E is positive.

4. (b) $\phi = BA = B \times \pi r^2$ (k = constant)

$\therefore \quad \phi \propto r^2 \Rightarrow \phi = kr^2$

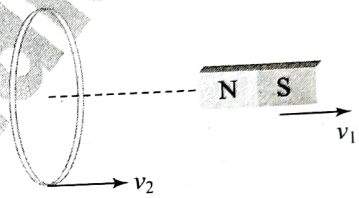
$\therefore \quad e = \frac{d\phi}{dt} = k \cdot 2r\frac{dr}{dt}$

From 0–1, r is constant, $\therefore \frac{dr}{dt} = 0$ hence, $e = 0$

From 1–2, $r = \alpha t$, $\therefore \frac{dr}{dt} = \alpha$ hence $e \propto r \Rightarrow e \propto t$

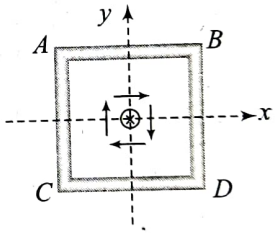
From 2–3, again r is constant, $\therefore \frac{dr}{dt} = 0$ hence $e = 0$

5. (a) Speed of the magnet



Relative speed between coil and magnet is zero, so there is no induced emf in the coil.

6. (d) Magnetic lines are tangential to the coil as shown in the figure. Thus net magnetic flux passing through the coil is always zero or the induced current will be zero.



7. (a) Crosses (×) linked with the loop are decreasing, so induced current in it is clockwise, i.e., from $B \to A$. Hence electrons flow from plate A to B, so plate A becomes positively charged.

8. (d) Induced charge $Q = -\frac{NBA}{R}(\cos\theta_2 - \cos\theta_1)$

$$= -\frac{NBA}{R}(\cos 180° - \cos 0°) \Rightarrow B = \frac{QR}{2NA}$$

9. (a) $dQ = \frac{d\phi}{R} = \frac{nA\,dB}{R} = \frac{100 \times 1 \times 10^{-3} \times 2}{10} = 2 \times 10^{-2}$ C

10. (a) $|dq| = \frac{d\phi}{R} = i\,dt$ = Area under i–t graph

$\therefore d\phi$ = (Area under i–t graph) R

$$= \frac{1}{2} \times 4 \times 0.1 \times (10) = 2 \text{ wb}$$

11. (a) $i = \frac{e}{R} = \frac{A}{R} \cdot \frac{dB}{dt} = \frac{(1 \times 10^{-2})^2}{16} \times 20 \times 10^{-3} = 1.25 \times 10^{-7}$ A

(Anticlockwise).

12. (c) For $t = 0$ to $t = 4$: $\frac{dB}{dt}$ is negative,

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and for $t = 4$ to $t = 8$: $\frac{dB}{dt}$ is positive.

$\therefore$ sign of current will change only once.

Multiple Correct Answers Type

13. (a, b, d)

$|e| = BA\omega \sin \omega t$

$\phi = Ba \cos \omega t$

$\therefore$ $|e|$ is maximum when $\omega t = \pi/2$

so ϕ is zero.

$\therefore$ $|e|$ is zero then ωt. So ϕ = is maximum.

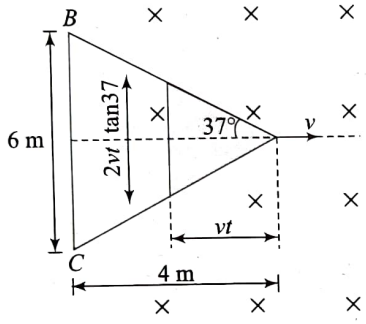
14. (b, c)

$$|e| = \left|-\frac{d\phi}{dt}\right| = |e| \alpha \left|-\frac{d\phi}{dt}\right|$$

and also area under the curve gives:

$\phi_i - \phi_i = \int d\phi = \int e \, dt$ change in flux.

15. (b, c)



Area of the loop inside magnetic field

$$A = \frac{1}{2} \times 6 \times 4 - \frac{1}{2} \times 2vt \tan 37° \times vt$$

$$\Rightarrow \quad A = 12 - \frac{3}{4}(vt)^2$$

$$\Rightarrow \quad \phi = BA = B\left(12 - \frac{3}{4}v^2t^2\right)$$

$$\Rightarrow \quad e = \frac{-d\phi}{dt} = +Bv^2\frac{3}{4} \times 2t = iR$$

Where R is equal to resistance of the loop and i is the current flowing in the loop

$$i = \frac{3Bv^2}{2R}t \qquad \text{hence } i \propto t$$

$$\Rightarrow \quad P \propto t^2 \text{ (parabolic variation)}$$

DPP 3.3

Single Correct Answer Type

1. (d) Both AD and BC are straight conductors moving in a uniform magnetic field and emf will be induced in both. This will cause electric fields in both, but no net current flows in the circuit.

2. (b) Emf induces across the length of the wire which cuts the magnetic field. (Length of c = Length d) > (Length of $a = b$). So $(e_c = e_d) > (e_a = e_b)$

3. (c) When loop is entering in the field, magnetic flux (*i.e.* ×) linked with the loop increases, so induced emf in it is $e = Bvl = 0.6 \times 10^{-2} \times 5 \times 10^{-2} = 3 \times 10^{-4}$ V (Negative).

When loop is completely entered in the field (after 5 sec), flux linked with the loop remains constant, so $e = 0$.

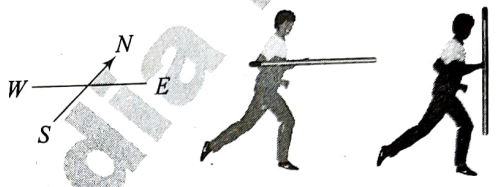
After 15 sec, loop begins to exit out, linked magnetic flux decreases so induced emf $e = 3 \times 10^{-4}$ V (Positive).

4. (b) If player is running with rod in vertical position towards east, then rod cuts the magnetic field of earth perpendicularly (magnetic field of earth is south to north).

Hence maximum emf induced is

$$e = Bvl = 4 \times 10^{-5} \times \frac{30 \times 1000}{3600} \times 3 = 1 \times 10^{-3} \text{ volt}$$

When he is running with rod in horizontal position, no field is cut by the rod, so $e = 0$.



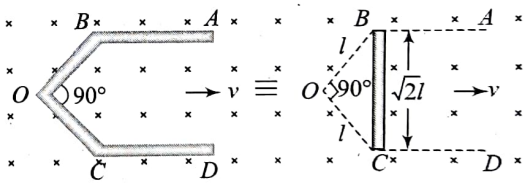
5. (b) Induced potential difference between two ends $= Blv = B_H lv$
$= 3 \times 10^{-5} \times 2 \times 50 = 30 \times 10^{-3}$ volt $= 3$ millivolt

By Fleming's right hand rule, end A becomes positively charged.

6. (b) Effective length between A and B remains the same.

7. (d) Induced emf $e = Bvl \Rightarrow e = Bv(2R) = \frac{2BvL}{\pi}$

8. (b) There is no induced emf in the part AB and CD because they are moving along their length while emf induced between B and C, i.e., between A and D can be calculated as follows:



Induced emf between B and C = Induced emf between A and B =

$$Bv(\sqrt{2}l) = 1 \times 1 \times 1 \times \sqrt{2} = 1.41 \text{ volt.}$$

9. (a) Induced current in the circuit $i = \frac{Bvl}{R}$

Magnetic force acting on the wire $F_m = Bil = B\left(\frac{Bvl}{R}\right)l$

$$\Rightarrow \quad F_m = \frac{B^2vl^2}{R}$$

External force needed to move the rod with constant velocity

$$(F_m) = \frac{B^2vl^2}{R} = \frac{(0.15)^2 \times (2) \times (0.5)^2}{3} = 3.75 \times 10^{-3} \text{ N}$$

10. (b) $P = Fv = Bil \times v = B\left(\frac{Bvl}{R}\right)l \times v = \frac{B^2v^2l^2}{R} \Rightarrow P \propto v^2$

It means the graph between P and v should be open up parabola.

Hence, (b) is correct.

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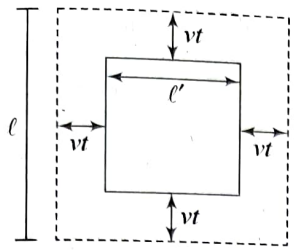
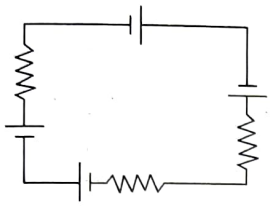
Comprehension Type

For Q. 11 to 15

11. (a) $E_{net} = 4Bl'v$



Equivalent Circuit:



$l' = l - 2vt$; $E_{net} = 4B(l - 2vt)v$

$E_{net} = 4Bv(l - 2vt)$

12. (b) $I = \frac{E_{net}}{4r} = \frac{4Blv}{4\lambda l} \Rightarrow I = \frac{Bv}{\lambda}$

13. (d) Force required on such wire = $Il'B$

$$\text{Force} = \frac{B^2v}{\lambda}(l - 2vt)$$

14. (c) Total power required to maintain constant velocity

$= p_{ele} = 4 \times E \times I = E_{net} \times I$

$$= 4Bv(\ell - 2vt) \times \frac{Bv}{\lambda} = \frac{4B^2v^2}{\lambda}(l - 2vt)$$

15. (a) Thermal power developed in circuit

$$= 4I^2r = 4I^2\lambda(l - 2vt) = 4\frac{B^2v^2}{\lambda^2}\lambda(l - 2vt)$$

Thermal power = $\frac{4B^2v^2}{\lambda}(l - 2vt)$

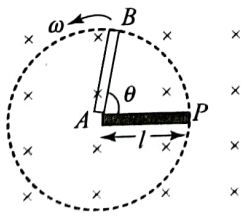
DPP 3.4

Single Correct Answer Type

1. (a) If in time t, the rod turns by an angle θ, the area generated by the rotation of rod will be

$= \frac{1}{2}l \times l\theta = \frac{1}{2}l^2\theta$

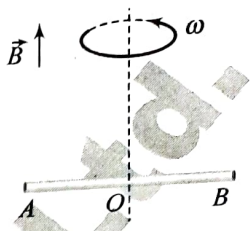
So the flux linked with the area generated by the rotation of rod



$$\phi = B\left(\frac{1}{2}l^2\theta\right)\cos 0 = \frac{1}{2}Bl^2\theta = \frac{1}{2}Bl^2\omega t$$

And so $e = \frac{d\phi}{dt} = \frac{d}{dt}\left(\frac{1}{2}Bl^2\omega t\right) = \frac{1}{2}Bl^2\omega$

2. (d) Potential difference between



O and A is $V_0 - V_A = \frac{1}{2}Bl^2\omega$

O and B is $V_0 - V_B = \frac{1}{2}Bl^2\omega$

so $V_A - V_B = 0$

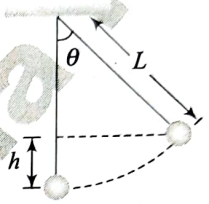
3. (c) By using $V = \frac{1}{2}Bl^2\omega$

For part AO ; $V_{OA} = V_O - V_A = \frac{1}{2}Bl^2\omega$

For part OC; $V_{OC} = V_O - V_C = \frac{1}{2}B(3l)^2\omega$

$\therefore V_A - V_C = 4Bl^2\omega$

4. (a)



$$\Rightarrow \quad h = L(1 - \cos\theta) \qquad \text{(i)}$$

Maximum velocity at equilibrium is given by

$$\therefore \quad v^2 = 2gh = 2g\,L(1 - \cos\theta) = 2g\,L\left(2\sin^2\frac{\theta}{2}\right)$$

$$\Rightarrow \quad v = 2\sqrt{gL}\sin\frac{\theta}{2}$$

Thus, maximum potential difference

$$V_{max} = BvL = B \times 2\sqrt{gL}\sin\frac{\theta}{2}L = 2BL\sin\frac{\theta}{2}(gL)^{1/2}$$

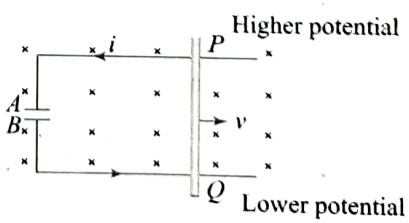
5. (b) Equivalent resistance of the given Wheatstone bridge circuit (balanced) is 3 Ω so total resistance in circuit is R = 3 + 1 = 4 Ω. The emf induced in the loop $e = Bvl$.

So induced current $i = \frac{e}{R} = \frac{Bvl}{R}$

$$\Rightarrow \quad 10^{-3} = \frac{2 \times v \times (10 \times 10^{-2})}{4} \Rightarrow v = 2 \text{ cm/sec}$$

6. (a) $Q = CV = C\,(Bvl) = 10 \times 10^{-6} \times 4 \times 2 \times 1 = 80\ \mu C$

According to Fleming's right- hand rule induced current flows from Q to P. Hence P is at higher potential and Q is at lower potential. Therefore A is positively charged and B is negatively charged.



7. (c) Motional emf $e = Bvl \Rightarrow e = 2 \times 2 \times 1 = 4$ V

This acts as a cell of emf E = 4 V and internal resistance $r = 2\ \Omega$.

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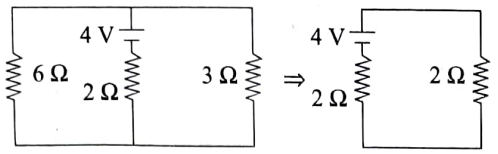
This simple circuit can be drawn as follows

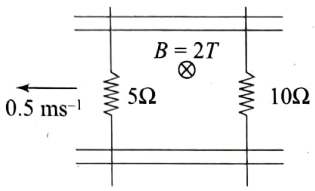


Current through the connector $i = \frac{4}{2+2} = 1\text{ A}$

$\therefore$ Magnetic force on connector

$F_m = Bil = 2 \times 1 \times 1 = 2\text{ N}$ (Towards left)

8. (d) When 5 Ω resistor is pulled left at 0.5 m/sec induced emf, in the said resistor = $e = vBl = 0.5 \times 2 \times 0.1 = 0.1$ V



Resistor 10 Ω is at rest so induced *emf* in it ($e = vBl$) be zero. Now net emf, in the circuit = 0.1 V and equivalent resistance of the circuit $R = 15\ \Omega$

Hence current $i = \frac{0.1}{15}\text{amp} = \frac{1}{150}\text{amp}$

And its direction will be anticlockwise (according to Lenz's law)

Comprehension Type

For Q. 9 to 12

9. (a), 10. (c), 11. (b), 12. (c)

When the speed is v, Emf = Blv

Resistance = $r + R$

Current = $\frac{Blv}{r+R}$

Force acting on the wire = ilB

$\frac{BlvlB}{R+r} = \frac{B^2l^2v}{R+r}$ [Force is opposite to velocity]

Acceleration on the wire = $\frac{B^2l^2v}{m(R+r)}$

The acceleration should be in the direction of force.

Hence, the acceleration should be in left direction.

As $a = \frac{dv}{dt} = \frac{-B^2l^2v}{m(R+r)}$

Hence $a = v\frac{dv}{dx} = \frac{-B^2l^2v}{m(R+r)}$ $\quad dx = \frac{(R+r)}{B^2l^2}dv$

On integrating, we get

$x = \frac{m(R+r)}{B^2l^2}$

For Q. 13 to 15

13. (a), 14. (b), 15. (c)

Net emf = $E - Bv\ell$

$i = \frac{E - Bvl}{r}$ (from b to a)

Force due to current,

$F = ilB = \left(\frac{E - Bv\ell}{r}\right).lB$

$= \frac{lB}{r}(E - Bvl)$

After some time When $E = Bv\ell$, then the wire moves in constant velocity v.

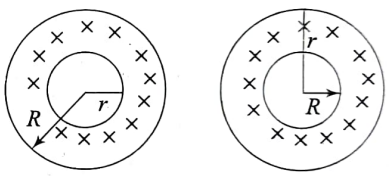
Hence $v = \frac{E}{B\ell}$

DPP 3.5

Single Correct Answer Type

1. (a) For $r < R$ $\oint E\,d\ell = A\frac{dB}{dt}$

$E2\pi r = (\pi r^2)\alpha \Rightarrow E = \frac{r\alpha}{2}$ or $E \propto r$



So, E–r graph is a straight line passing through origin.

At $\quad r = R, E = \frac{R\alpha}{2}$

For $\quad r > R, E2\pi r = (\pi R^2)\,\alpha$

$E = \frac{\alpha R^2}{2r}$ or $E \propto \frac{1}{r}$

2. (d) Perpendicular distance between BC and centre O is 10 cm.

Component of induced electric field along the rod $= \frac{d}{2}\frac{dB}{dt}$,

where d = perpendicular distance from centre to the rod. Hence, potential difference between the ends of rod.

$V = El = \ell \cdot \frac{d}{2}\frac{dB}{dt}$

$= \frac{10}{2} \times 10^{-2} \times 20 \times 10^{-2} 2 = 20\text{ mV}$

3. (c) Perpendicular distance between CD and O is 20 cm.

Therefore, induced emf in CD

$= \frac{d}{2}l\frac{dB}{dt} = \frac{20}{2} \times 10^{-2} \times 20 \times 10^{-2} \times 2 = 40\text{ mV}$

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Comprehension Type

4. (a) $e = A\dfrac{dB}{dt} = \pi a^2 B_0$

5. (c) $E = \dfrac{dV}{dr} = \dfrac{\pi a^2 B_0}{2\pi a} = \dfrac{1}{2} aB_0$

6. (a) $\alpha = \dfrac{\tau}{I} = \dfrac{q a^2 B_0}{2(ma^2)} = \dfrac{qB_0}{2m}$

7. (d) $P = Fv$, where $v = u + at$

Here, $u = 0$ and so $v = \dfrac{F}{m} t$

Thus, $P = \dfrac{F^2 t}{m} = \dfrac{q^2 B_0^2 a^2}{4m}$ at $t = 1$ s.

8. (b) At a distance r from centre,

$$e = A\frac{dB}{dt} = E.2\pi r, \text{ where } A = \pi r^2$$

Putting $r = R/2$, we get the option (b) as the answer.

9. (c) $F = qE$, where $E = \dfrac{r}{2}\dfrac{dB}{dt}$ as found in last part. Linear acceleration at any instant $a = F/m$ and $v = at$. Thus, $\omega = v/r$ can be calculated by substituting given values.

10. (c) $\dfrac{dB}{dt} = 6t^2 - 24$ which is zero at $t = 2$ s.

This implies that at $t = 2s$, induced emf and induced electric field is zero.

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CHAPTER 4

DPP 4.1

Subjective Type

1. (a) The mutual inductance of the coils is

$$M = \frac{N_B \Phi_{BA}}{i_A} = \frac{700\,(90.0 \times 10^{-6}\ \text{Wb})}{3.50\ \text{A}} = 18.0\ \text{mH}$$

(b) The inductance of coil A is

$$L_A = \frac{\Phi_A}{i_A} = \frac{400\,(300 \times 10^{-6}\ \text{Wb})}{3.50\ \text{A}} = 34.3\ \text{mH}$$

(c) The emf induced in the other coil is

$$|\varepsilon_B| = -M\frac{di_A}{dt} = (18.0\ \text{mH})(0.500\ \text{A/s})$$
$$= 9.00\ \text{mV}$$

2. (a) Solenoid S_1 creates a nearly uniform field everywhere inside it, given by $B_1 = \dfrac{\mu_0 N_1 i}{l}$

The flux through one turn of solenoid S_1,

$$\phi_2 = \frac{\mu_0 \pi R_2^2 N_1 i}{l}$$

The emf induced in solenoid S_2 is

$$e_2 = -\left(\frac{\mu_0 \pi R_2^2 N_1 N_2}{l}\right)\left(\frac{di}{dt}\right)$$

The mutual inductance, $M_{12} = \dfrac{\mu_0 \pi R_2^2 N_1 N_2}{l}$

(b) Solenoid S_2 creates a nearly uniform field everywhere inside it, given by $B_2 = \dfrac{\mu_0 N_1 i_2}{l}$ and nearly zero field outside. The flux through one turn of solenoid 1 is

$$\phi_1 = \frac{\mu_0 \pi R_2^2 N_2 i_2}{l}$$

The emf induced in solenoid 1,

$$e_1 = -\left(\frac{\mu_0 \pi R_2^2 N_1 N_2}{l}\right)\left(\frac{di_2}{dt}\right)$$

The mutual inductance, $M_{12} = \dfrac{\mu_0 \pi R_2^2 N_1 N_2}{l}$

(c) They are the same.

Single Correct Answer Type

3. (a) The self-induced emf at any instant is

$$\varepsilon_L = -L\frac{di}{dt}$$

Its average value is $\varepsilon_{L,\text{ave}} = -L\left(\dfrac{I_f - I_i}{t}\right)$

$$= (-2.00\ \text{H})\left(\frac{0 - 0.500\ \text{A}}{1.00 \times 10^{-2}\ \text{s}}\right)\left(\frac{\text{V.s/A}}{1\ \text{H}}\right) = +100\ \text{V}$$

4. (a) From $|\varepsilon| = L\left(\dfrac{\Delta i}{\Delta t}\right)$, we have

$$L = \frac{\varepsilon}{(\Delta i/\Delta t)} = \frac{25.0 \times 10^{-3}\ \text{V}}{10.0\ \text{A/s}} = 2.50 \times 10^{-3}\ \text{H}$$

From $L = \dfrac{N\Phi_B}{i}$, we have

$$\Phi_B = \frac{Li}{N} = \frac{(2.50 \times 10^{-3}\ \text{H})(4.00\ \text{A})}{500}$$
$$= 20\ \mu\text{T.m}^2$$

5. (b) From $|\varepsilon| = L\left(\dfrac{\Delta i}{\Delta t}\right)$, we have

$$L = \frac{|\varepsilon|}{|\Delta i/\Delta t|} = \frac{|\varepsilon||\Delta t|}{|\Delta i|} = \frac{(12.0 \times 10^{-3}\ \text{V})(0.500\text{s})}{|2.00\ \text{A} - 3.50\ \text{A}|}$$
$$= 4.00 \times 10^{-3}\ \text{H} = 4.00\ \text{mH}$$

6. (a) The emf is given by

$$\varepsilon = \varepsilon_0 e^{-kt} = -L\frac{di}{dt}$$

from which we obtain

$$di = -\frac{\varepsilon_0}{L}e^{-kt}dt$$

If we require $i \to 0$ as $t \to \infty$, the solution is

$$i = \frac{\varepsilon_0}{Lk}e^{-kt} = \frac{dq}{dt},\ \text{so}$$

$$Q = \int i dt = \int_0^\infty \frac{\varepsilon_0}{Lk}e^{-kt}dt = -\frac{\varepsilon_0}{Lk^2} \to |Q| = \frac{\varepsilon_0}{Lk^2}$$

7. (a) The current change is linear, so $\varepsilon = -L\dfrac{di}{dt} = -L\dfrac{\Delta i}{\Delta t}$.

$t = 0$ to 4 ms:

$$\varepsilon = -(4.00\ \text{mH})\frac{-2.00\ \text{mA}}{4.00\ \text{ms}} = +2.00\ \text{mV}$$

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t = 4 to 8 ms:

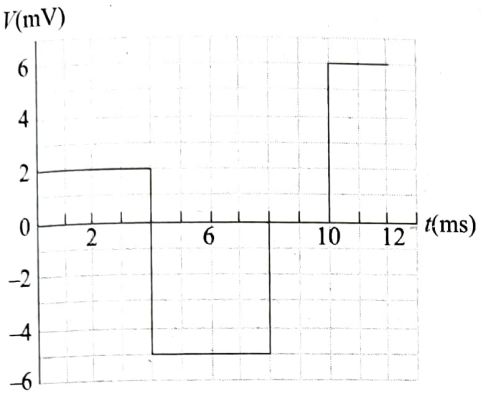
$$\varepsilon = -(4.00\text{ mH})\frac{+5.00\text{ mA}}{4.00\text{ ms}} = -5.00\text{ mV}$$

t = 8 to 10 ms:

$$\varepsilon = -(4.00\text{ mH})\frac{0}{2.00\text{ ms}} = 0.00\text{ mV}$$

t = 10 to 12 ms:

$$\varepsilon = -(4.00\text{ mH})\frac{-3.00\text{ mA}}{2.00\text{ ms}} = +6.00\text{ mV}$$



8. (d) We use equation, $|\varepsilon_2| = \left|-M\frac{di_1}{dt}\right|$, from which we obtain the mutual inductance:

$$M = \frac{|\varepsilon_2|}{|di_1/dt|} = \frac{0.0960\text{ V}}{1.20\text{ A/s}} = 0.080\text{ H} = 80\text{ mH}$$

9. (b) Let the changing current in coil 1 induce an emf in coil 2. Then,

$$\varepsilon_2 = -M\frac{di_1}{dt} = -(100\times10^{-6})\frac{d}{dt}[10.0\sin(1.00\times10^3 t)]$$

$$= -(100\times10^{-6})(10.0)(1.00\times10^3)\cos(1.00\times10^3 t)$$

$$= -(1.00)\cos(1.00\times10^3 t)$$

Therefore, the peak emf is $\varepsilon_{2\max} = 1.00$ V

10. (c) A current i in the large loop of radius R produces a magnetic field of magnitude $B = \frac{\mu_0 i}{2R}$ at its center. Since the radius of the small loop $r << R$, we may treat the flux through the small loop as being approximately $\Phi_B = BA\cos 0° = \frac{\mu_0 i}{2R}A = \frac{\mu_0 \pi r^2 i}{2R}$.

The mutual inductance of the loops is then

$$M = \frac{\Phi_B}{i} = \frac{\mu_0 \pi r^2}{2R}$$

Multiple Correct Answers Type

11. (a,b)

$$|\varepsilon| = L\frac{di}{dt} = \left|(90\times10^{-3})\frac{d}{dt}(1.0t^2 - 6.0t)\right|$$

$= (90)|(2.0t - 6.0)|$, ε is in millivolts (mV) and t in seconds.

at $t = 4.0$ s, $\varepsilon = 180$ mV

If $\varepsilon = (90)(2t - 6) = 0$ hence $t = 3.0$ s

Comprehension Type

12. (d) The inductance of the solenoid is

$$L = \frac{m_0 N^2 A}{l}$$

$$= \frac{(4\pi\times10^{-7}\text{ T.m/A})(400)^2[\pi(2.50\times10^{-2}\text{ m})^2]}{0.200\text{ m}}$$

$$1.97\times10^{-3}\text{ H} = 1.97\text{ mH} \approx 2\text{ mH}$$

13. (a) From $|\varepsilon| = L(\Delta I/\Delta T)$,

$$\frac{\Delta I}{\Delta t} = \frac{|\varepsilon|}{L} = \frac{75.0\times10^{-6}\text{ V}}{1.97\times10^{-3}\text{ H}}$$

$$= 38.0\times10^{-3}\text{ A/s} = 38.0\text{ mA/s}$$

14. (b) In terms of its cross-sectional area (A), length l, and number of turns (N), the self-inductance of a solenoid is given as $L = \mu_0 N^2 A/l$. Thus, for the given solenoid,

$$L = \frac{\mu_0 N^2(\pi d^2/4)}{l}$$

$$= \frac{(4\pi\times10^{-7}\text{ T.m/A})(1000)^2[\pi(8.00\times10^{-2}\text{ m})^2/4]}{4\pi^2\times10^{-2}}$$

$$= 16\times10^{-3}\text{ H} = 16\text{ mH}$$

15. (c) $|\varepsilon| = \left|-L\left(\frac{\Delta i}{\Delta t}\right)\right| = (16\times10^{-3}\text{ H})(+4.00\text{ A/s})$

$$= 64\times10^{-3}\text{ V} = 64\text{ mV}$$

DPP 4.2

Concept Based

1. (i) (a) The bulb glows brightly right away and then more and more faintly as the capacitor charges up. (b) The bulb gradually gets brighter and brighter, changing rapidly at first and then more and more slowly. (c) The bulb immediately becomes bright. (d) The bulb glows brightly right away and then more and more faintly as the inductor starts carrying more and more current (the inductor eventually acts as a short).

(ii) (a) The bulb goes out immediately because current stops immediately (charge ceases to flow). (b) The bulb glows for a moment as a spark jumps across the switch. (c) The bulb stays lit for a while, gradually getting fainter and fainter as the capacitor discharges through the bulb. (d) The bulb suddenly glows brightly. Then its brightness decreases to zero, changing rapidly at first and then more and more slowly.

Single Correct Answer Type

2. (a) $\because L \propto N^2 r$; $\frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2 \times \frac{r_1}{r_2}$

$$\Rightarrow \quad \frac{L}{L_2} = \left(\frac{1}{2}\right)^2 \times \left(\frac{r}{r/2}\right) = \frac{1}{2};\ L_2 = 2L$$

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3. (d) $e = B\cdot\frac{dA}{dt} = L\frac{di}{dt} \Rightarrow 1\times\frac{5}{10^{-3}} = L\times\frac{(2-1)}{2\times10^{-3}}$

$\Rightarrow \quad L = 10\text{ H}$

4. (c) Rate of decay of current between $t = 5$ ms to 6 ms $= \frac{di}{dt} = -$(slope of the line BC)

$$= -\left(\frac{5}{1\times10^{-3}}\right) = -5\times10^{3}\text{ A/s}$$

Hence induced emf

$$e = -L\frac{di}{dt} = -4.6\times(-5\times10^{3}) = 23\times10^{3}\text{ V}$$

5. (b) $e = -M\frac{di}{dt} = -1.5\frac{(1-0)}{(T/4)} = -\frac{6}{T}$, $T = \frac{2\pi}{\omega} = \frac{2\pi}{200} = \frac{\pi}{100}$

$\Rightarrow \quad |e| = \frac{600}{\pi} = 190.9\text{ V} \approx 191\text{ V}$

6. (c) According to i–t graph, in the first half current is increasing uniformly so a constant negative emf induces in the circuit.
In the second half current is decreasing uniformly so a constant positive emf induces.
Hence graph (c) is correct

7. (a) $\frac{di}{dt}$ = slope of i–t graph slope of graph (2) < slope of graph (1)

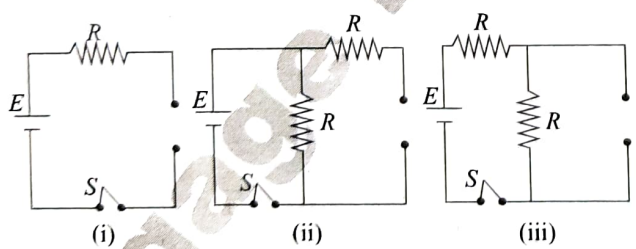
so $\left(\frac{di}{dt}\right)_2 < \left(\frac{di}{dt}\right)_1$

Also $L \propto \frac{1}{(di/dt)} \Rightarrow L_2 > L_1$

8. (c) At the time $t = 0$, e is max and is equal to E, but current i is zero.
As the time passes, current through the circuit increases but induced emf decreases.

9. (d) If at any instant, current through the circuit is i then applying Kirchhoff's voltage law, $iR + e = E \Rightarrow e = E - iR$. Therefore, graph between e and i will be a straight line having negative slope and having a positive intercept.

10. (b) At $t = 0$ current through L is zero so it acts as open circuit. The given figures can be redrawn as follow.



(i) (ii) (iii)

$i_1 = 0$ $\qquad i_2 = \frac{E}{R}$ $\qquad i_3 = \frac{E}{2R}$

Hence $i_2 > i_3 > i_1$.

11. (a) $U = \frac{1}{2}Li^2$

$\therefore \quad \text{Rate} = \frac{dU}{dt} = Li\left(\frac{di}{dt}\right)$

At $t = 0$, $I = 0$ $\therefore$ rate $= 0$

At $t = \infty$, $i = i_0$ but $\frac{di}{dt} = 0$, therefore rate $= 0$

12. (b) Applying Kirchhoff's law in wire is

$$V_A - iR - \frac{Ldi}{dt} - E = V_B$$

$$V_A - 1\times10 - 5\times10^{-3}(-10^{3}) - 10 = V_B$$

$$V_A - 10 + 5 - 10 = V_B$$

$$V_A - V_B = 15\text{ volt}$$

13. (b) At $t = 0$, inductor behave as open circuit so $i_1 = \frac{10}{10} = 1$ A

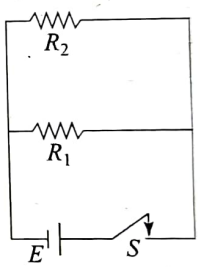
At $t = \infty$, inductor behaves as a short circuit, so $i_2 = \frac{10}{8} = \frac{5}{4}$ A

Hence, $\frac{i_1}{i_2} = \frac{1}{5/4} = \frac{4}{5} = 0.8$

Thus option (b) is correct.

14. (d) After a long time, steady current flows, so no emf is induced across inductor. Hence circuit reduces as shown.

So, current flowing $i = \frac{E(R_1 + R_2)}{R_1 R_2}$



15. (c) The current at time t is given by $i = i_0(1 - e^{-t/\tau})$

Where $i_0 = \frac{E}{R}$ and $\tau = \frac{L}{R}$

So $\qquad dq = i\,dt = i_0(1 - e^{-t/\tau})\,dt$

$$Q = \int dQ = \int_0^{\tau} i_0(1 - e^{-t/\tau})\,dt$$

$$= i_0\left\{\left[\int_0^{\tau} dt - \int_0^{\tau} e^{-t/\tau}\,dt\right]\right\} = \frac{i_0\tau}{e}$$

$\Rightarrow \quad Q = \frac{\frac{E}{R}\cdot\frac{L}{R}}{e} = \frac{EL}{eR^2} \Rightarrow Q = \frac{EL}{eR^2}$

DPP 4.3

Concept Based

1. When the capacitor is fully discharged, the current in the circuit is a maximum. The inductance of the coil is making the current continue to flow. At this time the magnetic field of the coil

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contains all the energy that was originally stored in the charged capacitor. The current has just finished discharging the capacitor and is proceeding to charge it up again with the opposite polarity.

2. The energy stored in a capacitor is proportional to the square of the electric field, and the energy stored in an induction coil is proportional to the square of the magnetic field. The capacitor's energy is proportional to its capacitance, which depends on its geometry and the dielectric material inside. The coil's energy is proportional to its inductance, which depends on its geometry and the core material. The capacitor's energy is proportional to the charge it stores, the coil's energy is proportional to the current it holds.

Subjective Type

3. (a) $P = i\Delta V = (3.00\text{ A})\,(22.0\text{ V}) = 66.0\text{ W}$

(b) $P = i\Delta V_R = i^2R = (3.00\text{ A})^2\,(5.00\ \Omega) = 45.0\text{ W}$

(c) We treat the real inductor as an ideal inductor (with no resistance) in series with an ideal resistor (with no inductance). When the current is 3.00 A, Kirchhoff's loop rule reads

$$+22.0\text{ V} - (3.00\text{ A})(5.00\ \Omega) - \Delta V_L = 0$$
$$\Delta V_L = 7.00\text{ V}$$

The power being stored in the inductor is

$$i\Delta V_L = (3.00\text{ A})(7/00\text{ V}) = 21.0\text{ W}$$

(d) The power supplied by the battery is equal to the sum of the power delivered to the internal resistance of the coil and the power stored in the magnetic field.

(e) Yes.

(f) Just after $t = 0$ the current is very small, so the power delivered to the internal resistance of the coil (iR^2) is nearly zero, but the rate of the change of the current is large, so the power delivered to the magnetic field (Ldi/dt) is large, and nearly all the battery power is being stored in the magnetic field. Long after the connection is made, the current is not changing, so no power is being stored in the magnetic field, and all the battery power is being delivered to the internal resistance of the coil.

Single Correct Answer Type

4. (b) When the switch has been closed for a long time, battery, resistor, and coil carry constant current $I_i = \frac{\varepsilon}{R}$. When the switch is opened, current in battery and resistor drops to zero, but the coil carries this same current for a moment as oscillations begin in the LC loop. We interpret the problem to mean that the voltage amplitude of these oscillations is ΔV, in $\frac{1}{2}C(\Delta V)^2 = \frac{1}{2}LI_i^2$.

Then, $L = \frac{C(\Delta V)^2}{I_i^2} = \frac{C(\Delta V)^2}{\varepsilon^2}$

5. (a) At different times, $(U_C)_{max} = (U_L)_{max}$, so

$$\left[\frac{1}{2}C(\Delta V)^2\right]_{max} = \frac{1}{2}LI_i^2$$

Then, $I_i = \sqrt{\frac{C}{L}}(\Delta V)_{max} = \sqrt{\frac{1.00\times10^{-6}\text{ F}}{10.0\times10^{-3}\text{ H}}}(40.0\text{ V})$

$= 0.400\text{ A} = 400\text{ mA}$

6. (b) Equivalent inductance $L_{eq} = L + 2L = 3L$

$C_{eq} = C + 2C = 3C$

$$f = \frac{1}{2\pi\sqrt{L_{eq}C_{eq}}} = \frac{1}{6\pi\sqrt{LC}}$$

Frequency of oscillation

7. (d) Initially the capacitor provide zero resistance. Therefore, initial current is $E = 2R$.

Finally the capacitor provides infinite resistance, therefore final current is $E/2(R + r)$, assuming r be the small resistance provided by the inductor.

Hence the best option is that after a long time interval current through battery will be less than current through it initially.

Multiple Correct Answers Type

8. (a,b,c,d)

From Equation, $i = \frac{\varepsilon}{R}(1 - e^{-Rt/L})$:

(a) The maximum current, after a long time t, is $i = i_1 = \frac{\varepsilon}{R} =$ 2.00 A.

At that time, the inductor is fully energized and

$P = i(\Delta V) = (2.00\text{ A})\,(10.0\text{ V}) = 20.0\text{ W}$

(b) $P_{lost} = i^2R = (2.00\text{ A})^2\,(5.00\ \Omega) = 20.0\text{ W}$

(c) The inductor has no resistance: $P_{inductor} i(\Delta V_{drop}) = 0$

(d) $U_B = \frac{1}{2}Li^2 = \frac{1}{2}(10.0\text{ H})(2.00\text{ A})^2 = 20.0\text{ J}$

9. (b, c, d)

(a) The frequency of oscillation of the circuit is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{\sqrt{(0.100\text{ H})(1.00\times10^{-6}\text{ F})}} = 503\text{ Hz}$$

(b) The maximum charge on the capacitor is

$$Q = C\varepsilon = (1.00\times10^{-6}\text{ F})\,(12.0\text{ V}) = 12.0\ \mu\text{C}$$

(c) To find the maximum current I, we equate

$$\frac{1}{2}C\varepsilon^2 = \frac{1}{2}LI_i^2$$

Then solve for I, to obtain

$$I_i = \varepsilon\sqrt{\frac{C}{L}} = 12.0\text{ C}\sqrt{\frac{1.00\times10^{-6}\text{ F}}{0.100\text{ H}}} = 37.9\text{ mA}$$

(d) The total energy the circuit possesses at t = 3.00s and at all times is

$$U = \frac{1}{2}C\varepsilon^2 = \frac{1}{2}(1.00\times10^{-6}\text{ F})(12.0\text{ V})^2 = 72.0\ \mu\text{J}$$

Comprehension Type

10. (a) The two resistors are in parallel. Their resistance values 500 Ω and R are related to their equivalent resistance R_{eq} by

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$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{500\,\Omega}$$

and the equivalent resistance is related to the time constant of the circuit by

$$\tau = \frac{L}{R_{eq}} \Rightarrow \frac{1}{R_{eq}} = \frac{\tau}{L}$$

Thus, $\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{500\,\Omega} = \frac{\tau}{L}$

Solving for R,

$$\frac{1}{R} = \frac{\tau}{L} - \frac{1}{500\,\Omega} = \frac{10.0\times10^{-6}\,\text{s}}{4.00\times10^{-3}\,\text{H}} - \frac{1}{500\,\Omega}$$

which gives $R = 2\times10^{3}\,\Omega = 2.0\,\text{k}\Omega$

11. (b) The current will immediately begin to die from the value it had just before the switch was thrown to position b. Before the switch position was changed, the current was constant in time, so there was no emf induced in the inductor. The current was just

$$i = \frac{\Delta V}{R_{eq}} = \Delta V\frac{\tau}{L} = (24\,\text{V})\frac{10.0\times10^{-6}\,\text{s}}{4.00\times10^{-3}\,\text{H}}$$

$$= 60\times10^{3} = 60\,\text{mA}$$

For Q. 12 and 13

For the increasing current $i = \frac{\varepsilon}{R}(1 - e^{-Lt/R})$. The final value is $\frac{\varepsilon}{R}$, so the condition on Δt is

$$0.800\frac{\varepsilon}{R} = \frac{\varepsilon}{R}(1 - e^{-L\Delta t/R})$$

$$e^{-L\Delta t/R} = 0.200$$

$$e^{+L\Delta t/R} = 5.00$$

$$\frac{L\Delta t}{R} = \ln 5.00$$

$$\Delta t = \frac{R\ln 5.00}{L}$$

At the moment when the battery is removed, the current in the coil is quite precisely $\frac{\varepsilon}{R}$. During the decrease, $i = I_i e^{-Lt/R} = \frac{\varepsilon}{R}e^{-Lt/R}$.

12. (a) At $t = \Delta t = \frac{R\ln 5.00}{L}$,

$$\frac{i}{I_i} = e^{-L\Delta t/R} = 0.200 = 20.0\%$$

13. (c) at $t = 2\Delta t$,

$$\frac{i}{I_i} = e^{-L2\Delta t/R} = (e^{-L\Delta t/R})^2$$

$$= (0.200)^2 = 0.040\,0 = 4.0\%$$

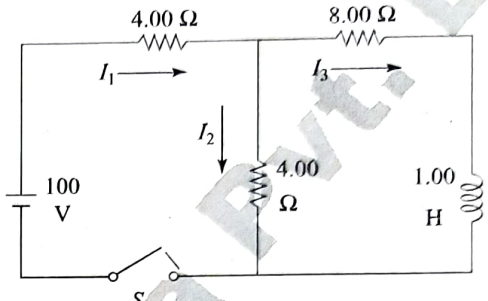
For Q. 14 and 15

Name the currents as shown in the figure. By Kirchhoff's laws:

$$i_1 = i_2 + i_3 \qquad \text{(i)}$$

$$+10.0\,\text{V} - 4.00i_1 - 4.00i_2 = 0 \qquad \text{(ii)}$$

$$+10.0\,\text{V} - 4.00i_1 - 8.00i_3 - (1.00)\frac{di_3}{dt} = 0 \qquad \text{(iii)}$$



From (i) and (ii),

$$+10.0 - 4.0i_1 - 4.0i_1 + 4.0i_3 = 0$$

$$i_1 = 0.5i_3 + 1.25\,\text{A}$$

Then [iii] becomes

$$10.0\,\text{V} - 4.0(0.5\,i_3 + 1.25\,\text{A}) - 8.0i_3 - (1.0)\frac{di_3}{dt} = 0$$

$$(1.0\,\text{H})\left(\frac{di_3}{dt}\right) + (10.0\,\Omega)i_3 = 5.0\,\text{V}$$

or $$5.0\,\text{V} - (10.0\,\Omega)i_3 - (1.0\,\text{H})\left(\frac{di_3}{dt}\right) = 0 \qquad \text{(iv)}$$

Equation (iv) can be compared to the general form $\varepsilon - iR - L\frac{di}{dt} = 0$, which has the solution $i = \frac{\varepsilon}{R}(1 - e^{-Rt/L})$.

Thus, we have:

14. (b) $$i_3 = \left(\frac{5.0\,\text{V}}{10.0\,\Omega}\right)[1 - e^{-(10.0\,\Omega)t/1.0\,\text{H}}]$$

$$= (0.5\,\text{A})[1 - e^{-10t/s}]$$

15. (d) $i_1 = 1.25 + 0.5i_3 = 1.5\,\text{A} - (0.25\,\text{A})e^{-10t/s}$

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CHAPTER 5

DPP 5.1

Single Correct Answer Type

1. (c) $\nu = \frac{\omega}{2\pi} = \frac{120 \times 7}{2 \times 22} = 19$ Hz

$V_{r.m.s.} = \frac{240}{\sqrt{2}} = 120\sqrt{2} \approx 170$ V

2. (b) $E = E_0 \cos \omega t = E_0 \cos \frac{2\pi t}{T}$

$= 10 \cos \frac{2\pi \times 50 \times 1}{600} = 10 \cos \frac{\pi}{6} = 5\sqrt{3}$ V

3. (d) Peak value to r.m.s. value means, current becomes $\frac{1}{\sqrt{2}}$ times.

So from $i = i_0 \sin 100\,\pi t \Rightarrow \frac{1}{\sqrt{2}} \times i_0 = i_0 \sin 100\pi t$

$\Rightarrow \quad \sin \frac{\pi}{4} = \sin 100\,\pi t \Rightarrow t = \frac{1}{400}$ sec $= 2.5 \times 10^{-3}$ sec

4. (c) Phase difference $\Delta\phi = \phi_2 - \phi_1 = \frac{\pi}{6} - \left(\frac{-\pi}{6}\right) = \frac{\pi}{3}$

5. (c) $\overline{i^2} = \frac{\int i^2 dt}{\int dt} = \frac{\int_2^4 (4t)\,dt}{\int_2^4 dt} = \frac{4\int_2^4 t\,dt}{2}$

$= 2\left[\frac{t^2}{2}\right]_2^4 = [t^2]_2^4 = 12$

$\Rightarrow \quad i_{rms} = \sqrt{\overline{i^2}} = \sqrt{12} = 2\sqrt{3}$ A

6. (b) 1. rms value $= \frac{x_0}{\sqrt{2}}$

2. $x_0 \sin \omega t \cos \omega t = \frac{x_0}{2} \sin 2\omega t \Rightarrow$ rms value $= \frac{x_0}{2\sqrt{2}}$

3. $x_0 \sin \omega t + x_0 \cos \omega t \Rightarrow$ rms value $= \sqrt{\left(\frac{x_0}{\sqrt{2}}\right)^2 + \left(\frac{x_0}{\sqrt{2}}\right)^2}$

$= \sqrt{x_0^2} = x_0$

7. (c) $i_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} = \frac{T^2}{\sqrt{5}}$

8. (a) $V_{rms} = \sqrt{\frac{1}{T}\int_0^T 10^2 dt} = 10$ V

9. (c) $I_{av} = \frac{\int_0^{T/2} i\,dt}{\int_0^{T/2} dt} = \frac{\int_0^{T/2} I_0 \sin(\omega t)\,dt}{T/2}$

$= \frac{2I_0}{T}\left[\frac{-\cos \omega t}{\omega}\right]_0^{T/2} = \frac{2I_0}{T}\left[\frac{\cos\left(\frac{\omega T}{2}\right)}{\omega} + \frac{\cos 0^o}{\omega}\right]$

$= \frac{2I_0}{\omega T}[-\cos \pi + \cos 0^o] = \frac{2I_0}{2\pi}[1+1] = \frac{2I_0}{\pi}$

10. (b) (1) For time interval $0 < t < T/2$

$I = kt$, where k is the slope

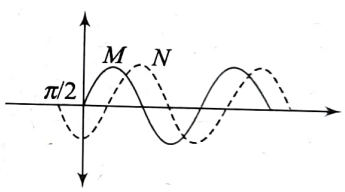
For inductor as we know, induced voltage $V = -L\frac{di}{dt}$

$\Rightarrow \quad V_1 = -KL$

(2) For time interval $\frac{T}{2} < t < T$

$t = -Kt \Rightarrow V_2 = KL$

11. (b) From the graph shown below. It is clear that phase lead of N over M is $-\frac{\pi}{2}$. Since time period (*i.e.* taken to complete one cycle) = 0.4 sec. Hence frequency $\nu = \frac{1}{T} = 2.5$ Hz



12. (d) In purely inductive circuit, voltage leads the current by 90°.

13. (c) $X_L = 2\pi f L \Rightarrow X_L \propto f \Rightarrow \frac{1}{X_L} \propto \frac{1}{f}$

i.e., graph between $\frac{1}{X_L}$ and f will be a hyperbola.

14. (c) We have $X_C = \frac{1}{C \times 2\pi f}$ and $X_L = L \times 2\pi f$

15. (b) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$, i.e., $X_C \propto \frac{1}{f}$

DPP 5.2

Single Correct Answer Type

1. (a) Yes, in AC if branch AB has R, BC has a capacitor C, and BD has a pure inductance L

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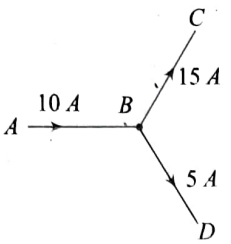


2. (c) I_L lags behind I_R by a phase of $\frac{\pi}{2}$, while I_C leads by a phase of $\frac{\pi}{2}$.

3. (a) For dc, $R = \frac{V}{i} = \frac{100}{1} = 100\ \Omega$

For ac, $Z = \frac{V}{i} = \frac{100}{0.5} = 200\ \Omega$

$\because \quad Z = \sqrt{R^2 + (\omega L)^2} \Rightarrow 200 = \sqrt{(100)^2 + 4\pi^2 (50)^2 L^2}$

$\therefore \quad L = 0.55$ H

4. (b) This is because, when frequency v is increased, the capacitive reactance $X_C = \frac{1}{2\pi v C}$ decreases and hence the current through the bulb increases

5. (b) $X_L = R, X_C = R/2$

$\therefore \quad \tan\phi = \frac{X_L - X_C}{R} = \frac{R - \frac{R}{2}}{R} = \frac{1}{2}$

$\Rightarrow \quad \phi = \tan^{-1}(1/2)$

Also $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \frac{R^2}{4}} = \frac{\sqrt{5}}{2} R$

6. (c) At A: $X_C > X_L$
At B: $X_C = X_L$
At C: $X_C < X_L$

7. (d) For the first circuit $i = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$

$\therefore$ Increase in ω will cause a decrease in i.

For the second circuit $i = \frac{V}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$

$\therefore$ Increase in ω will cause an increase in i.

8. (a) $R = 6 + 4 = 10\ \Omega$

$X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\ \Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{2000 \times 50 \times 10^{-6}} = 10\ \Omega$

$\therefore \quad Z = \sqrt{R^2 + (X_L - X_C)^2} = 10\ \Omega$

Amplitude of current $= i_0 = \frac{V_0}{Z} = \frac{20}{10} = 2$ A

9. (b) When C is removed, circuit becomes RL circuit. Hence,

$\tan\frac{\pi}{3} = \frac{X_L}{R}$ (i)

When L is removed, circuit becomes RC circuit. Hence,

$\tan\frac{\pi}{3} = \frac{X_C}{R}$ (ii)

From equation (i) and (ii) we obtain $X_L = X_C$. This is the condition of resonance and in resonance $Z = R = 100\ \Omega$

10. (b) Resonance frequency

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{8 \times 10^{-3} \times 20 \times 10^{-6}}} = 2500$ rad s^{-1}

Resonance current $= \frac{V}{R} = \frac{220}{44} = 5$ A

11. (b) Since voltage is lagging behind the current, so there must be no inductor in the box.

12. (d) The current will lag behind the voltage when reactance of inductance is more than the reactance of condenser. Thus,

$\omega L > \frac{1}{\omega C}$ or $\omega > \frac{1}{\sqrt{LC}}$

or $\quad n > \frac{1}{2\pi\sqrt{LC}}$ or $n > n_r$ where n_r = resonant frequency.

Comprehension Based

13. (a) Let at an instant $V_R = (V_R)_m \cos\omega t$

$\therefore \quad 2 = 4\cos\omega t$

$\cos\omega t = 1/2$

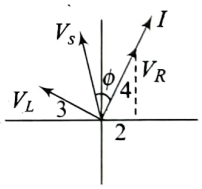
$\therefore \quad \cos\omega t = 60°$

Since V_L is 90° ahead of V_R

$V_L = (V_L)_m \cos(\omega t + 90)$

$= -3\sin\omega t = -3\sin 60° = -3\cos 30°$

$|(V_L)_m| = 3\cos 30°$



14. (b) From phasor diagram

$(V_S)_m = \sqrt{(V_R)_m^2 + (V_L)_m^2} = 5$ V

$\tan\phi = \frac{(V_L)_m}{(V_R)_m} = \frac{3}{4} \quad \therefore \quad \phi = 37°$

$\therefore \quad |V_S| = |(V_S)_m \cos(\omega t + 37°)|$

$= 5|\cos(60° + 37°)| = 5|\cos 97°| = 5\cos 83°$

15. (a) From phasor diagram it is clear that when instantaneous current will decrease, the magnitude of potential difference will increase.

DPP 5.3

Single Correct Answer Type

1. (c) $i_{WL} = i_{rms}\sin\phi \Rightarrow \sqrt{3} = 2\sin\phi \Rightarrow \sin\phi = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \phi = 60°$ so p.f. $= \cos\phi = \cos 60° = \frac{1}{2}$

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2. (c) $Z=\sqrt{R^2+\left(\frac{1}{2\pi\nu C}\right)^2}$

$$=\sqrt{(3000)^2+\frac{1}{\left(2\pi\times 50\times\frac{2.5}{\pi}\times 10^{-6}\right)^2}}$$

$$\Rightarrow \quad Z=\sqrt{(3000)^2+(4000)^2}=5\times 10^3\ \Omega$$

So power factor $\cos\phi=\frac{R}{Z}=\frac{3000}{5\times 10^3}=0.6$ and power

$$P=V_{rms}i_{rms}\cos\phi=\frac{V_{rms}^2\cos\phi}{Z}\Rightarrow P=\frac{(200)^2\times 0.6}{5\times 10^3}=4.8\ \text{W}$$

3. (c) With dc: $P=\frac{V^2}{R}\Rightarrow R=\frac{(10)^2}{20}=5\ \Omega$

With ac: $P=\frac{V_{rms}^2R}{Z^2}\Rightarrow Z^2=\frac{(10)^2\times 5}{10}=50\ \Omega^2$

Also $Z^2=R^2+4\pi^2\nu^2L^2$

$\Rightarrow 50=(5)^2+4(3.14)^2\nu^2(10\times 10^{-3})^2\Rightarrow \nu=80$ Hz

4. (b) $P=\frac{1}{2}V_0i_0\cos\phi\Rightarrow 1000=\frac{1}{2}\times 200\times i_0\cos 60°$

$$\Rightarrow \quad i_0=20\ \text{A}\Rightarrow i_{rms}=\frac{i_0}{\sqrt{2}}=\frac{20}{\sqrt{2}}=10\sqrt{2}\ \text{A}$$

5. (c) $P=E_{rms}i_{rms}\cos\phi=\frac{E_0}{\sqrt{2}}\times\frac{i_0}{\sqrt{2}}\times\frac{R}{Z}$

$$\Rightarrow \quad \frac{E_0}{\sqrt{2}}\times\frac{E_0}{Z\sqrt{2}}\times\frac{R}{Z}\Rightarrow P=\frac{E_0^2R}{2Z^2}$$

Given $X_L=R$ so, $Z=\sqrt{2}R\Rightarrow P=\frac{E_0^2}{4R}$

6. (d) $\tan\phi=\frac{X_L}{R}=\frac{X_C}{R}\Rightarrow\tan 60°=\frac{X_L}{R}=\frac{X_C}{R}$

$$\Rightarrow \quad X_L=X_C=\sqrt{3}R$$

i.e., $\quad Z=\sqrt{R^2+(X_L-X_C)^2}=R$

So average power $P=\frac{V^2}{R}=\frac{200\times 200}{100}=400$ W

7. (d) The voltage V_L and V_C are equal and opposite so voltmeter reading will be zero.

Also $R=30\ \Omega$, $X_L=X_C=25\ \Omega$

So $i=\frac{V}{\sqrt{R^2+(X_L-X_C)^2}}=\frac{V}{R}=\frac{240}{30}=8$ A

8. (d) $Z=\sqrt{(R)^2+(X_L-X_C)^2}$;

$R=10\ \Omega$, $X_L=\omega L=2000\times 5\times 10^{-3}=10\ \Omega$

$X_C=\frac{1}{\omega C}=\frac{1}{2000\times 50\times 10^{-6}}=10\ \Omega$, i.e., $Z=10\ \Omega$

Maximum current $i_0=\frac{V_0}{Z}=\frac{20}{10}=2$ A

Hence $i_{rms}=\frac{2}{\sqrt{2}}=1.4$ A

and $V_{rms}=4\times 1.41=5.64$ V

9. (a) At angular frequency ω, the current in RC circuit is given by

$$i_{rms}=\frac{V_{rms}}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \qquad \text{(i)}$$

Also $\frac{i_{rms}}{2}=\frac{V_{rms}}{\sqrt{R^2+\left(\frac{1}{\frac{\omega}{3}C}\right)^2}}=\frac{V_{rms}}{\sqrt{R^2+\frac{9}{\omega^2C^2}}}$ (ii)

From equation (i) and (ii) we get

$$3R^2=\frac{5}{\omega^2C^2}\Rightarrow\frac{\frac{1}{\omega C}}{R}=\sqrt{\frac{3}{5}}\Rightarrow\frac{X_C}{R}=\sqrt{\frac{3}{5}}$$

10. (b) In RC series circuit voltage across the capacitor leads the voltage across the resistance by $\frac{\pi}{2}$.

Comprehension Based

11. (c) Current drawn is maximum at resonant angular frequency.

$L_{eq}=4\ mH$ $C_{eq}=10\ \mu$F

$$\omega=\frac{1}{\sqrt{LC}}=5000\ \text{rad/s}$$

12. (d) C_{eq} decreases, thereby increasing resonant frequency.

13. (b) At resonance $i_{rms}=\frac{100}{100}=1$ A

Power supplied $=V_{rms}I_{rms}\cos\phi(\phi=0)$ at resonance

$0\ P=100$ W

14. (b) Average energy stored $=\frac{1}{2}Li_{rms}^2$

$$=\frac{1}{2}(2.4\times 10^{-3}\ \text{H.})(1\ \text{A})^2=1.2\ \text{mJ}$$

15. (d) As 1 μs time duration is very less than time period T at resonance, thermal energy produced is not possible to calculate about the start of the given time duration without information.

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